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Aids to Computation

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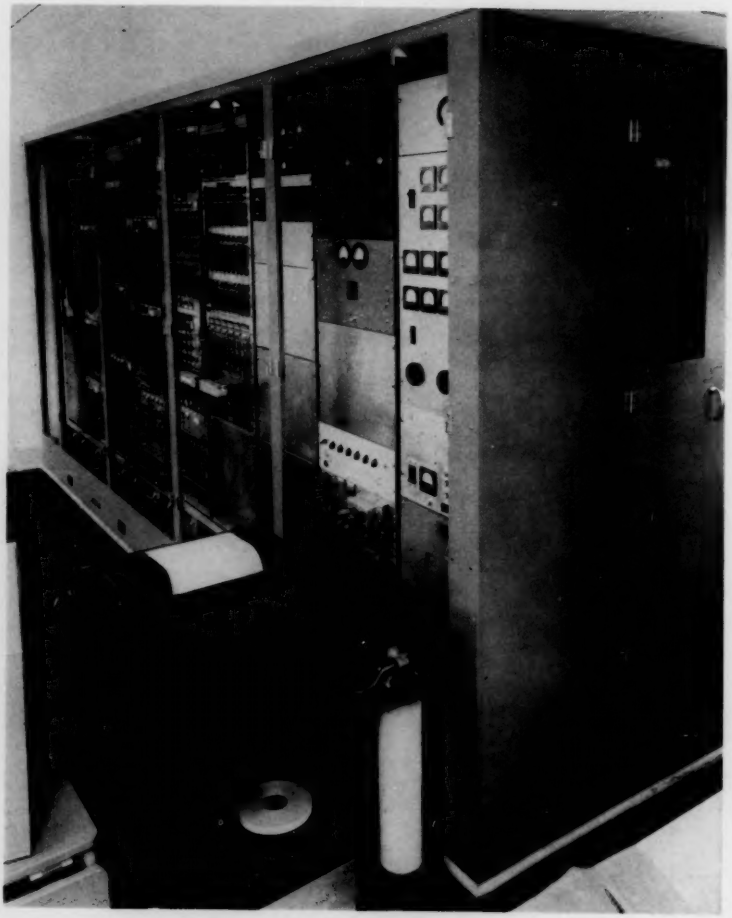

THE SEAC

A Guide to Tables on Punched Cards

A list of tables on punched cards, compiled by W. J. ECKERT, was published in the October 1945 issue of *MTAC* [v. 1, p. 433–436]. Since then, the number of laboratories interested in mathematical computations has greatly increased and as a consequence many new and important tables have become available.

Computing techniques, too, have undergone radical changes with the advent of the electronic "C.P.C." of the IBM Corporation and the new electronic multipliers which are accessible to laboratories of moderate size. Then there are the large, high-speed digital machines which employ punched cards; they are continually adding punched card tables of general interest.

In a sense, the need for the more elementary punched card tables, such as square roots, reciprocals, and squares has decreased—at least for those laboratories that have electronic multipliers. In fact, several of the larger laboratories that formerly kept files of elementary tables on punched cards have discarded them. There are, however, a considerable number of smaller laboratories that still need the elementary functions, and even the laboratories of moderate size would prefer a tested, available table of the Riemann Zeta function, say, to generating needed values with equipment currently at their command. In addition, "optimum interval" tables [*MTAC*, v. 1, p. 173–176] have found favor in some quarters. A number of tables of this character have been added since 1945.

When, at the request of the editors of *MTAC*, the present authors undertook to bring the list of punched card tables up-to-date, it soon became apparent that the wealth of new material made it advisable to revise the guide completely.

It was felt that classification of material according to subject matter rather than location would be most helpful to the laboratory that needed a certain table. In this type of classification it would have been difficult to include the file number which a given source attached to a table. Such information is therefore omitted. If a required table is adequately described, even a large and busy laboratory will probably find it quite easy to locate its file. An exception was made in the case of the UMT depository, that with time may acquire much heterogeneous material which might be hard to locate without a precise file number.

Quite often the user of a table may want to know the original source of the keypunched values; whether such values were taken from well-known books, or whether the laboratory computed the table on punched card equipment from first principles. Moreover, one would like to know whether the possessor of a table checked it, or acquired it from another laboratory and did not get around to testing its accuracy. The authors tried to elicit this information. In the case of many tables of the higher mathematical functions, they knew from their own experience where such tables originated. For the elementary tables, the statements of the reporting laboratories were taken at face value, without exhaustive checks. Memory for such information becomes nebulous over the years, and many laboratories probably failed to

get credit for their own work; perhaps a few were credited for efforts they did not make. It is hoped that readers will report any such errors when they notice them.

The tables reported are all "unclassified." Some of the largest laboratories reported only a small fraction of their tables of general interest. Since such laboratories no doubt have good and sufficient reasons for limiting the library they wish to make available to others, the authors made no attempt to list tables which the source did not specifically mention. However, most tables of general interest are eventually published and reported in *MTAC* and private correspondence with the laboratories involved may bring fruitful results in such cases.

The authors acknowledge gratefully the cooperation of the following members of laboratories who submitted lists of their tables: M. ABRAMOWITZ, H. H. AIKEN, P. ARMER, W. D. BELL, J. BELZER, J. M. BOERMEESTER, E. C. BOWER, C. C. BRAMBLE, S. R. BRINKLEY, Jr., B. F. CHEYDLEUR, G. M. CLEMENCE, R. P. COATES, C. F. DAVIS, W. J. ECKERT, B. FERBER, A. D. FRANKLIN, C. C. GOTLIEB, R. W. HAMMING, C. HASTINGS, Jr., R. HENSEL, F. H. HOLLANDER, C. C. HURD, J. P. KELLY, E. C. KENNEDY, W. A. KITTS, 3d, H. KRAFT, D. H. LEHMER, M. LOTKIN, E. LUNDT, H. MCALLISTER, A. OPLER, H. POLACHEK, G. W. REITWIESNER, C. V. RUZEK, J. SCHILT, V. SCHOMAKER, J. SHERMAN, C. T. TAI and F. M. VERZUH. Many of them went to considerable trouble in answering correspondence with the authors and in checking the listing of the items reported by them. The difficult task of preparing the typed manuscript from a preliminary card index was performed by Mrs. KATHARINE WARREN. She not only checked the typed data with painstaking care, but also gave much attention to maintaining uniformity in style and format.

To the extent that this guide reflects accurately the tables available, credit is due to the wholehearted cooperation of all the laboratories involved. The authors of this guide alone bear the responsibility for any errors of their commission in reporting the data, and they will appreciate information about any inaccuracies discovered by readers.

The meaning of "sources" and the symbols employed are explained below.

SOURCES

The source of the original tables from which card entries were key-punched is indicated wherever it is known. A list of numbered "sources" with the addresses at time of publication of this guide is given below. Some of these are laboratories which retain key-punched values. Others, such as (66) and (40), are books of tables or other texts to which reference is made in the guide. Some laboratories, like (64), no longer have the cards credited to them. In such cases, the place where the tables have been deposited is indicated alongside the name. Reference details about tables or texts were sometimes taken from (40) without further verification.

MEANING OF SYMBOLS

The symbol $x=0(.01)5(.1)8$; 5D or 5S means as usual that the argument ranges from 0 to 5 at intervals of 0.01, from 5 to 8 at intervals of 0.1, and that the entries are given to 5 decimals (5D) or to 5 significant figures (5S).

The symbol [(41)–(9), (7)] in the source column indicates that the entries are based on values in source (41), and that both (9) and (7) key-punched them independently. Similarly [(41), (9)–(9)] means that all key-punching was done by (9); that some entries were taken from (41) and others were computed by (9). The symbol [(9)] implies that computing and key-punching were done by (9).

The laboratories that possess a given punched card table are listed in the column headed "Available at." Some laboratories have stated that they have verified, mathematically or otherwise, the accuracy of the tables. This fact is indicated by the letter C following the source number.

The symbol $\Delta^k f$ (or merely Δ^k) indicates the k^{th} forward difference of the function; $\delta^{2k} f$ (or δ^{2k}) indicates the central difference of order $2k$. Similarly $\delta^k f_i$ and $\Delta^k f_i$ will sometimes be abbreviated to δ_i^k, Δ_i^k, respectively.

CLASSIFICATION

The classification followed is that adopted by FLETCHER, MILLER, & ROSENHEAD in *An Index of Mathematical Tables* [source (40)]. However, since the number of tables listed here is not very large, many sub-classifications have been omitted. For example, the trigonometric functions are listed as follows:

7.1 Tables with argument in radians
7.2 Tables with argument in decimals of a degree
7.3 Tables with argument in centesimal measure (grades or parts of the complete circle)
7.4 Tables with argument in degrees, minutes, and seconds

On the other hand, sub-headings such as 7.11, 7.12 have usually been omitted.

When many similar tables are listed under one heading, the grouping is according to the number of decimals or significant figures given in the entries. For example: all tables given to 5 decimals or significant figures will be followed by those given to 6 decimals, and so forth.

Some items seemed to belong to classes not specifically listed in (40). The following additional classes have been added:

25. Random numbers and functions corresponding to random arguments
26. Tables relating to aerodynamics. A wide variety of specialized tables, all of them available on punched cards, are listed in the voluminous unclassified reports issued by (54). Mention of the existence of such tables is made, but too much space would be required to include them in this guide. The reader interested in aerodynamic tables may consult the reports distributed by (54). Some of the punched cards are available at (6).
27. Tables relating to number theory
28. Tables relating to astronomy
29. Actuarial tables. Here, too, no specific details are given. Complete details can be obtained from the Society of Actuaries or from the IBM Corp., 590 Madison Ave., New York, N. Y.
30. Tables relating to map projections and geodesy

Some items belong equally well to more than one category. Thus 2^n belongs under (2.3) as a table of powers and under (27) as a power of the prime 2. In this instance the table has been indexed in both categories. In other cases, where a listing is given in (40) the listing is usually included only under the category assigned to the item in (40), even though another choice might have been more desirable from a logical viewpoint.

An effort was made to unify the entries as far as possible. On the other hand, two very similar tables may differ in minor details which might be important to the laboratory operator. For the most extensive table of a type is not necessarily the most desirable one; a compact table may often be preferred to a large table, given to a higher accuracy. For this reason, tables which differed sufficiently in format or other details are listed separately. This is especially true in the case of tables of trigonometric functions, and in the tables of powers. In this connection, it was not always possible to indicate in one item all the data available on one card. For example, some laboratories have the functions x^1, $1/x$, x^2, and x^3 on one card. On account of classification difficulties, these are listed separately.

In the case of optimum interval tables, the number of cards comprised in the compilation has been indicated, when this information was available to the authors. Unless otherwise indicated, such tables are interpolable linearly.

SOURCES

1 Bell Telephone Laboratories, Murray Hill Laboratory, Murray Hill, N. J.
2 Wm. D. Bell, Telecomputing Corp., 133 East Santa Anita Ave., Burbank, Cal.
3 California Inst. of Tech., Cooperative Wind Tunnel, Pasadena 4, Cal.
4 Central Statistical Laboratory, Union Carbide and Carbon Corp., Oak Ridge, Tenn.
5 Computation Laboratory, Harvard University, Cambridge 38, Mass.
6 Consolidated Vultee Aircraft Corp., Ordnance Aerophysics Laboratory, Daingerfield, Texas
7 Consolidated Vultee Aircraft Corp., San Diego, Cal.
8 Dow Chemical Co., Western Division, P.O. Box 351, Pittsburg, Cal.
9 E. C. Bower, Engineering Department, Douglas Aircraft Corp., Santa Monica, Cal.
10 General Electric Co., Apparatus Dept., Schenectady 5, N. Y.
11 Massachusetts Inst. of Tech., Cambridge, Mass. (Office of Statistical Services)
12 North American Aviation Corp., att: Charles F. Davis, Tabulating Unit, International Airport, Los Angeles, Cal.
13 Ohio State University, Cryogenic Laboratory, Columbus 10, Ohio
14 Rand Corporation, 1500 Fourth St., Santa Monica, Cal.
15 The Texas Company, Research Laboratories, Beacon, N. Y.
16 University of Toronto, Computation Center, Toronto, Ontario, Canada
17 Thos. J. Watson Astronomical Computing Bureau, Columbia Univ., New York, N. Y.
18 Watson Scientific Computing Laboratory, Columbia Univ., New York, N. Y.

19 Central Computing Service, Univ. of Cal., Berkeley 4, Cal., att: P. L. Morton, Cory Hall
20 California Institute of Technology, Gates & Crellin Lab. of Chemistry, Pasadena, Cal.
21 Ballistic Research Laboratories, Aberdeen Proving Ground, Aberdeen, Md.
22 U. S. Naval Observatory, Washington 25, D. C.
23 National Bureau of Standards, Computation Lab., Washington 25, D. C.
24 National Bureau of Standards, Inst. for Numerical Analysis, Los Angeles 24, Cal.
25 U. S. Dept. of Interior, Bureau of Mines, Pittsburgh 13, Pa.
26 U. S. Naval Ordnance Laboratory, White Oak, Silver Spring 19, Md.
27 U. S. Naval Proving Ground, Dept. of Computation and Ballistics, Dahlgren, Va.
28 U. S. Army Map Service, Corps of Engineers, Washington 16, D. C.
30 Society of Actuaries, % Service Bureau, IBM Corp., 590 Madison Ave., New York, N. Y.
40 FLETCHER, MILLER & ROSENHEAD, *An Index of Mathematical Tables*. London, Scientific Computing Service, Ltd., 23 Bedford Square, W. C. 1, 1946
41 *Barlow's Tables of Squares, Cubes, Square Roots, Cube Roots and Reciprocals*. 4th ed., London and New York, 1944
42 JOHN TODD, *Table of Arctangents of Rational Numbers*. NBS Applied Math. Series, no. 11, March 1951
43 *Marchant Methods*. Marchant Calculating Machine Co., New York, N. Y.
44 *Vega's Logarithmic Tables of Numbers and Trigonometric Functions*. New York, D. Van Nostrand Co., 1912
45 *Mathematical Tables from Handbook of Chemistry and Physics*. 9th ed., compiled by CHARLES D. HODGMAN, Cleveland, Ohio, Chemical Rubber Co., 1948
46 EDWIN CHAPPELL, *Five-Figure Mathematical Tables*. 1915, reprinted by D. Van Nostrand, New York
47 H. ANDOYER, *Nouvelles Tables Trigonométriques Fondamentales*, v. 1, 3, Paris, Hermann, 1915, 1918
48 G. N. WATSON, *A Treatise on the Theory of Bessel Functions*. 2nd ed., New York, Macmillan, 1944
49 L. SCHWARZ, *Luftfahrtforschung*. V. 20, no. 12, p. 341–372, 1944
50 K. HAYASHI, *Sieben-und mehrstellige Tafeln der Kreis- und Hyperbelfunktionen*. Berlin, Springer, 1926
51 F. CALLET, *Tables Portatives de Logarithmes*. Paris, F. Didot, 1795 (Tirage, 1819). (sub-tabulated by E. C. BOWER)
52 J. J. ÅSTRAND, *Hülfstafeln zur leichten und genauen Auflösung des Kepler'schen Problems*. Leipzig, Engelmann, 1890
53 J. C. P. MILLER, *Tables of $I_n(x)$, $K_n(x)$, and $Y_n(x)$*. The University Math. Laboratory, Cambridge, England. To be published for the Royal Society (the last of the British Assn. for the Adv. Sc. tables). Typed manuscript made available by author to (23).
54 Applied Physics Laboratory, Johns Hopkins University, Silver Spring, Md. *Handbook of Supersonic Aerodynamics*, Jan., 1949. Reports can

be purchased from Supt. of Documents, Gov't Printing Office, Washington, D. C. Vols. 1 and 2 now available.

55 SAMUEL HERRICK, *Tables for Rocket and Comet Orbits.* Punched cards available at (24). Functions tabulated: $C_e(U)$, $S_e(U)$, $X_e(U)$; $C_h(U)$, $S_h(U)$, $X_h(U)$, where

(a) $U = E - \sin E$; $C_e(U) = 1 - \cos E$; $S_e(U) = \sin E$; $X_e(U) = E$ for $0 < U \leq 3.15$

(b) $U = \sinh F - 1$; $C_h(U) = \cosh F - 1$; $S_h(U) = \sinh F$; $X_h(U) = F$ for $0 < U \leq 300$

Especially useful for finding the functions corresponding to very small values of U.

56 C. CRANZ, *Lehrbuch der Ballistik.* V. 1, Berlin, Springer, 1925

57 Applied Mathematics Panel, National Defense Research Committee, AMP *Report* 24–1, Sept. 1943, submitted by MERRILL M. FLOOD, Princeton Univ.

58 HAROLD T. DAVIS, *Tables of the Higher Mathematical Functions.* V. 2, Principia Press, Bloomington, Indiana, 1935

59 R. L. ANDERSON & E. E. HOUSEMAN, *Tables of Orthogonal Polynomial Values Extended to n = 104. Research Bull.* 297, Agricultural Exp. Station, Statistical Section, Iowa State College, Ames, Iowa, 1942

60 D. H. LEHMER: (a) Tables of Ramanujan's $\tau(n)$, reported in *MTAC*, UMT **101**, v. 4, p. 162, July, 1950

(b) Table of the sums of fifth powers of the divisors of n, $n = 1(1)5000$, reported in *MTAC*, UMT **114**, v. 5, p. 28, Jan., 1951

61 National Bureau of Standards, *Tables Relating to Mathieu Functions.* New York, Columbia Univ. Press, 1951

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64 Dept. of Engineering Research, U.C.L.A. [tables available at (24)]

65 H. W. HOLTAPPEL, *Tafels van e^x.* Groningen, Holland, Noordhoff, 1938

66 British Assoc. for the Advancement of Science, Mathematical Tables, London, England, Cambridge Univ. Press

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68 J. PETERS, *Seven-Place Values of Trigonometric Functions for every Thousandth of a Degree.* New York, D. Van Nostrand, 1942 ed.

69 AMELIA DE LELLA, *Five-Place Table of Natural Trigonometric Functions to Hundredths of a Degree.* New York, John Wiley and Sons, 1934

70 U. S. Lake Survey, Military Grid Unit, Dept. of Engineers [cards at (23)]

71 ARNOLD N. LOWAN & JACK LADERMAN, "Table of Fourier Coefficients," *Jn. Math. Physics*, v. 22, 1943, p. 136–147

72 H. J. GRAY, R. MERWIN & J. G. BRAINERD, Solution of the Mathieu Equation, *Am. Inst. Elec. Engrs.*, v. 67, 1948, p. 429–441

73 UMT FILE, (Unpublished Mathematical Tables) % D. H. Lehmer. (When writing for cards, mention UMT reference number.)

74 R. M. ROBINSON, (a) *Stencils for the Solution of Systems of Linear Congruences Modulo* 2. See *MTAC*, UMT 120, v. 5, p. 85. (b) *Table of Integer Solutions of* $|y^2 - x^3| < x$. See *MTAC*, UMT 125, v. 5, p. 162.
75 J. BAUSCHINGER & J. PETERS, *Logarithmisch-trigonometrische Tafeln mit acht Dezimalstellen*. V. 1, second ed., Leipzig, Engelmann, 1936
76 Royal Observatory at Greenwich, *Nautical Almanac and Astronomical Ephemeris*, London, H. M. Stationery Office, London W. C. 2
77 U. S. Navy, Hydrographic Office, *Tables of Computed Altitude and Azimuth*, Publication No. 214, Washington, D. C., U. S. Government Printing Office, 1940
78 The University Mathematical Laboratory, University of Cambridge, Cambridge, England
79 C. E. VAN ORSTRAND, *Tables of the Exponential Function and of the Circular Sine and Cosine to Radian Argument*. Nat'l Academy of Sciences, 5th memoir, v. 14, Washington, U. S. Government Printing Office, 1921
80 H. BRANDENBURG, *Siebenstellige trigonometrische Tafel* . . . . Leipzig, Lorentz, 1931
81 P. WIJDENES, *Five-place Tables*. Groningen, Noordhoff
82 H. M. Nautical Almanac Office (L. J. Comrie), *Planetary Coordinates for the Years* 1800 *to* 1940. London, H. M. Stationery Office, 1933

Source	*Available at*		*Description of Tables*
		2.	Powers, real and complex; simple rational functions of powers (Also see 6.19)
[(21)]	(11)C, (16)C, (21)C, (24)	**2.1**	x^2: $x = 0(.0001)0.9999$; exact On same card with x^3, x^4, x^5 at (24)
[(18)]	(18)C		x^2: $x = 0(.01)2.25$; 2D; on same card with $2x^2$, x^3, x^4
[—(3)]	(16), (24)C		x^2, Δ: $x = 1(1)1000$; exact; on same card with $\sqrt{x}$, $\sqrt{10x}$, $1/x$
	(16), (24)C		x^2: $x = 1(1)9999$ Δ, δ^2; exact; on same card with $1/x$ Available at (17)C for $x \geq 1000$.
[(41)–(27)]	(27)		x^2: $x = 1(1)10{,}000$; exact; on same card with x^3.
[(41)–(18)]	(18)C		x^2: $x = 1(1)9999$; 7D; on same card with $1/x$, $1/x^2$
[(41)–(21)]	(21)		x^2: $x = 1(1)12{,}500$; exact
	(23)C		x^2: $x = 10(10)10{,}000$; 7S or 8S On same card with x^k, $k = -1, -2, -3, 3, \frac{1}{2}, \frac{3}{2}, \frac{1}{3}$
[(18)]	(18)C		$2x^2$: $x = 0(.01)2.25$; 2D; on same card with x^2, x^3, x^4
[(21)]	(16)C, (21), (24)	**2.2**	x^3: $x = 0(.0001)0.9999$; exact; on same card with x^2, x^4, x^5

Source	*Available at*		*Description of Tables*
[(18)]	(18)C		x^2: $x = 0(.01)2.25$; 2D; on same card with x^2, $2x^2$, x^4
[(41)–(27)]	(27)		x^3: $x = 1(1)10{,}000$; exact; on same card with x^2
[(21)]	(21)		x^3: $x = 1(1)12{,}500$; exact
[(23)]	(11)C, (23)C		x^3: $x = 10(10)10{,}000$; 7S or 8S
[(18)]	(18)C	2.3	x^4: $x = 0(.01)2.25$; 2D; on same card with x^2, $2x^2$, x^3
[(11)]	(11)C		x^4: $x = 1(1)2500$; 8S
[(41)–(27)]	(27)		x^4: $x = 0(1)10{,}000$; exact
[(21)]	(16)C, (21), (24)		x^4, x^5: $x = 0(.0001)0.9999$; exact
[(16)]	(16)C		x^6, x^7: $x = 0(.0001)0.9999$; exact
[(18)]	(18)C		x^p: $x = 0(.001)0.999$; $p = 2(1)6$; 8D
[(26)]	(26)C		2^n; $n = 1(1)100$; exact
[(22)]	(11)C, (22)C	2.4	$1/x$, Δ: $x = 1(.001)4(.01)9.99$; 6D
[(18)]	(13), (18)C		$1/x$: $x = 0.1$(opt.int.)3; 6S; 1896 cards
[(18)]	(10), (18)C		$1/x$: $x = 0.629$(opt.int.)16.15; 6S; 1196 cards
[(14)]	(14)C		$1/x$: $x = 1$(opt.int.)10.00810; 5S or 6S; 577 cards
	(16)		$1/x$: $x = 2(.01)6.6$; 7D; on same card with $\log_e x$
[(41)–(21)]	(21)		$1/x$; A, B; $A = \frac{1}{2}(\Delta_i + \Delta_{i-1})$, $B = \frac{1}{2}\delta^2$ $x = 0.1(.0001)0.9999$; 7S
[—(3)]	(16), (24)C		$1/x$, $-\Delta$: $x = 1(1)1000$; 7D; on same card with $\sqrt{x}$, $\sqrt{10x}$, x^2
[(41)–(18)]	(18)C		$1/x$: $x = 1(1)9999$; 7D; on same card with $1/x^2$, x^2
[(41)–(27)]	(27)		$1/x$, $\Delta(1/x)$: $x = 1(1)10{,}000$ for $1/x$; $x = 1000(1)10{,}000$ for $\Delta(1/x)$; 7S
[(41)–(21)]	(21)		$1/x$: $x = 1(1)12{,}500$; 7S
[(41)–(26)]	(23)C, (24)		$1/x$, $-\Delta$: $x = 1000(1)2000(2)4000$-(5)10,000; 7S; on same card with $\sqrt{x}$, $\sqrt{10x}$
[(23)]	(23)C		$1/x$: $x = 10(10)10{,}000$; 7S or 8S
[(41)–(11)]	(11)C		$1/x$: $x = 1(.1)20$ and $x = 1(1)2500$; 8D
[(23)]	(16), (24)C		$1/x$: $x = 1(1)9999$; Δ, δ^2; 10D; on same card with x^2
[(17)]	(17)C		$1/x$: $x = 1000(1)9999$; 10D; on same card with x^2

Source	*Available at*		*Description of Tables*
[(26)]	(26)		$1/x$, Δ: x = 1000(1)19,999; 10D
[(41)–(18)]	(18)C	2.5	$1/x^2$: x = 1(1)9999; 7D; on same card with $1/x$, x^2
[(23)]	(23)C		x^{-2}, x^{-3}: x = 10(10)10,000; 7S or 8S
[—(3)]	(16), (24)C	2.6	$\sqrt{x}$, $\sqrt{10x}$, Δ: x = 1(1)1000; 4D
[(25)]	(25)		$\sqrt{x}$: x = 1(opt.int.)100; 5S; 401 cards
	(14)C		$\sqrt{x}$: x = 1(opt.int.)100; 5D; 1082 cards
[(43)–(13)]	(13)C, (18)		$\sqrt{x}$: x = 1(opt.int.)100; 6D; 332 cards
[(18)]	(18)C		$\sqrt{x}$: x = 0(opt.int.)0.9950; 6D; on same card with $\sqrt{1-x}$; 583 cards
[(18)]	(18)C		$\sqrt{x}$: x = 0.5(opt.int.)2; 6D; 246 cards
	(24)		$\sqrt{x}$: x = 1(opt.int.)99.430; 6D; 181 cards
[(18)]	(18)C		$\sqrt{x}$: x = 1(opt.int.)100; 6D; 1793 cards
[(18)]	(18)C		$\sqrt{x}$: x = 1(opt.int.).0001; to about 10D; 242 cards
[(21)]	(23)C, (24)		$\sqrt{x}$, $\sqrt{10x}$, Δ: x = 1000(1)2000(2)4000-(5)10,000; 6D
[(41)–(21)]	(21)		$\sqrt{x}$, $\sqrt{10x}$: x = 1(1)12,500; 7D for $x < 1000$, 6D for $x > 1000$
[(26)]	(23), (26)		$\sqrt{x}$, Δ, δ^2, $\frac{1}{2}(\Delta_0 + \Delta_{-1})$: x = 101-(1)10,000; 6D Range extended by (23) to include x = 1(1)100.
[(26)]	(26)		$\sqrt{x}$, $\sqrt{10x}$, Δ: x = 1000(1)1995; 2000-(2)4000(5)9995; 6D
[(26)]	(23)C, (26)		$\sqrt{x}$, Δ: x = 10,000(10)100,000; 6D
[(23)]	(23)C		$\sqrt{x}$, $\sqrt{10x}$: x = 1000(1)10,000; 8D in $\sqrt{x}$, 7D in $\sqrt{10x}$
[(2)]	(24)		$\sqrt{x}$, Δ, $1/\sqrt{x}$, Δ: x = 1(1)9999; 6D in $\sqrt{x}$, 8D in $1/\sqrt{x}$ [last place inaccurate]; on same card with $1/x$, $\Delta(1/x)$
[(41)–(27)]	(27)		$1/\sqrt{x}$: x = 1(1)1000; 7D
	(24)		$1/\sqrt{x}$: x = 1.000(opt.int.)99.180; 7D; 902 cards
[(41)–(21)]	(21)C		$\sqrt{x}$, A, B; $A = \frac{1}{2}(\Delta_i + \Delta_{i-1})$, $B = \frac{1}{2}\delta_i^2$: x = 0.0001(.0001)1(.001)9.999; 8D
[(11)]	(11)C		$\sqrt{x}$: x = 0(1)1200; 8D
[(41)–(27)]	(27)		$\sqrt{x}$, $\sqrt{10x}$, Δ; x = 1,000(1)10,000; 8S–9S; also $\sqrt{x}$, x = 1(1)1000
[(23)]	(24)C		$\sqrt{x}$, Δ, δ^2: x = 1(1)1000; 10S

Source	*Available at*		*Description of Tables*
[(23)]	(24)C		$\sqrt{x}$, $\sqrt{10x}$; Δ, δ^2: $x = 1000(1)10{,}000$; 10S
			Tables for reducing interferometer data
[(26)]	(26)		$f(x, y) = (\sqrt{(x+1)^2 - y^2} - \sqrt{x^2 - y^2})/(2x + 1)$
			$x, y = 1(1)100$; $y \leq x$; 5D
[(18)]	(18)C		Table of $f = \sqrt{1 - x}$: $x = 0$(opt.int.)-0.9950; 6D; 583 cards
			On same card with $\sqrt{x}$
[(26)]	(26)		$(1 - x^2)^{-\frac{1}{2}}$: $x = 0(.005)0.1(.001)-0.808(.0005)0.9$; 4D to 6D
			$x = 0.9(.0001)0.96-(.00001)0.99499$; 5D to 7D
[(23)]	(23)C		$(1 - x^2)^{-\frac{1}{2}}$: $x = 0$(various)1; 8D
[(26)]	(23)C	2.63	$x^{\frac{1}{2}}$: $x = 10(10)10{,}000$; 7S or 8S
[(82)–(17)]	(17), (18)		$x^{-\frac{1}{2}}$: $x = 2(.001)7.5(.01)20$; 8D
[(25)]	(25)	2.7	$x^{\frac{1}{2}}$: $x =$ 1(opt.int.)1000; 5S; 540 cards
[(26)]	(23)C, (24)		$x^{\frac{1}{2}}$, Δ: $x = 1{,}000(1)2{,}000(2)4{,}000(5)-9{,}999$; 6D
			At (26)C with slight modifications.
[(41)–(21)]	(21)		$x^{\frac{1}{2}}$, A, B; $A = \frac{1}{2}(\Delta_i + \Delta_{i-1})$, $B = \frac{1}{2}\delta^2$; $x = 1(.001)9.999$; 7D
[(41)–(21)]	(21), (24)		$x^{\frac{1}{2}}$: $x = 1(1)12{,}500$; 7D for $x < 1000$; 6D for $x > 1000$
[(26)]	(23)C		$x^{\frac{1}{2}}$: $x = 10(10)10{,}000$; 7S or 8S
[(41)–(27)]	(27)		$x^{\frac{1}{2}}$: $x = 1(1)10{,}000$; also Δ for $x >$ 1000; 8S
[(23)–(24)]	(16), (18), (23), (24)C		$x^{\frac{1}{2}}$, $(10x)^{\frac{1}{2}}$, $(100x)^{\frac{1}{2}}$: $x = 0(.01)10$; 9D or 10D
[(23)–(24)]	(18), (23), (24)C		$x^{\frac{1}{2}}$, $(10x)^{\frac{1}{2}}$, $(100x)^{\frac{1}{2}}$: $x = 0(.01)10$; 9D or 10D
[(23)–(24)]	(24)C		$x^{\frac{1}{2}}$, $(10x)^{\frac{1}{2}}$, $(100x)^{\frac{1}{2}}$: $x = 0(.01)10$; 15D in $x^{\frac{1}{2}}$; 14D in $(10x)^{\frac{1}{2}}$ and $(100x)^{\frac{1}{2}}$
[(23)–(24)]	(24)C		$x^{\frac{1}{2}}$, $(10x)^{\frac{1}{2}}$, $(100x)^{\frac{1}{2}}$: $x = 0(.01)10$; 15D
[(18)]	(18)C	2.8	$x^{1/6}$: $x = 0.1$(opt.int.)1; 6D; 284 cards
[(67)–(24)]	(24)C		$x^{\frac{1}{2}}$, $x^{\frac{1}{4}}$, $x^{-\frac{1}{4}}$, $x^{-\frac{1}{2}}$: $x = 1(.01)10(.1)100(1)1000(10)10{,}000$; 10S
[(13)]	(13)C	2.81	$10^{-10^a}10^{a^2}$, $a = 0, 1, 2$: $x = -6.99-(.01)0.99$; 7D
		2.9	Powers of complex numbers

Source	*Available at*		*Description of Tables*
[(16)]	(16)C		$\sqrt{w^2 + i} = a + ib$: $w = 0(.01)17$; 8D in a; 7D in b
[(24)]	(24)C		$(x + iy)^n = u_n + iv_n$: $x = -20(2)(-10)(1)0$; $y = 0(1)20(10)400$; $n = 0(1)10$; exact
[(10)]	(10)	2.96	$1/z$: $z = x + iy$; $x, y = -4(.02)4$; 5D
		3.11	Factorials
[(41)–(27)]	(27), (28)		$x!$: $x = 0(1)100$; 8S
[(21)]	(21)		$x!$ and $1/x!$: $x = 1(1)1000$; 62S; Computed on the ENIAC in connection with determination of e
		3.3	Binomial coefficients
[(26)]	(26)		$n = 1(1)100$; exact
		4.4	Modified Bernoulli polynomials
[(24)]	(24)C		$b_k(x) = \sum_{n=1}^{\infty} n^{-k} \sin [nx + \frac{1}{2}(k-1)\pi]$, $k \geq 1$ $= [(-1)^k \frac{1}{2}(2\pi)^k B_k(\frac{1}{2}x/\pi)]/k!$, where B_k is the Bernoulli polynomial: $x = \pi y/36$, $y = 0(1)36$; $k = 1(1)11$; 17D
		6	Common logarithms and antilogarithms
[(18)]	(18)C	6.1	$\log_{10} x$: $x = 1(1)99$; 3D
[(23)]	(23)C		$\log_{10} x$: $x = 1(1)13{,}000(10)23{,}000$; 5D
[(45)–(4)]	(4)		$\log_{10} x$: $x = 1(.001)9.999$; 5D
[(18)]	(10), (18)C		$\log_{10} x$: $x = 1$(opt.int.)$10(10^i)$; $i = -1(1)5$; 5D; 1463 cards
[(13)]	(13)C, (18)		$\log_{10} x$: $x = 1$(opt.int.)10; 6D; 568 cards
[(23)]	(23)C		$\log_{10} x$, Δ: $x = 10(10)10{,}000$; 7S or 8S
[(44)–(27)]	(27)		$\log_{10} x$: $x = 1000(1)10{,}000$; 7D
[(44)–(21)]	(21)		$\log_{10} x$: $x = 1(.0001)9.9999$; 7D
[(64)]	(16), (24)		$\log_{10} x$: $x = 1000$(opt.int.)$10{,}000$; 7D; 586 cards
[(64)]	(24)		$\log_{10} x$: $x = 0$(opt.int.)0.962400; for quadratic interpolation; 7D; 119 cards
[(18)]	(26)		$\log_{10} x$: $x = 1$(opt.int.)9.9958; 8D
[(18)]	(10), (16), (18)C		$\log_{10} x$: $x = 1$(opt.int.)10; 8D; 5640 cards

Source	*Available at*		*Description of Tables*
[(64)]	(16), (24)		$\log_{10} x$: $x = 0$(opt.int.)0.998284; 8D; 707 cards
[(75)–(2)]	(2)C, (18)C		$\log_{10} x$, Δ: $x = 1(.0001)10$; 8D. (Prepared at Vega Aircraft Corp. by RAY CRAWFORD, in collaboration W. D. BELL.)
[(75)–(2)]	(2)C, (24)C		$\log_{10} x$, Δ, $1/\Delta$: $x = 1(.001)9.999$; 8D
		6.2	Radix table
[(18)]	(18)C		$f = 10^{\pm z}$; $m \cdot 10^n$, $m = 1(1)9$; $n = 1(1)4$; to about 7S; 9 cards
[(8)]	(8)	6.26	$\log_{10}\left(\frac{1-x}{y-x}\right)$: $x = 0.02(.01)0.99$; $y = 0(.005)0.05(.01)0.2$; 5D
		6.4	Antilogarithms
[(8)]	(8)		10^x: $x = 0$(opt.int.)0.999; 5D; 407 cards
[(46)–(21)]	(21)		10^x: $x = 0(.0001)1$; 5D
[(44)–(22)]	(22)C		10^x: $x = 0(.001)0.999$; 8D; auxiliary functions for interpolation
[(23)–(14)]	(14)C		10^x, Δ: $x = 0(.00001)1$; 8D
[(64)]	(24)		10^x: $x = 0$(opt.int.)-5.3467875; 9D; 707 cards
[(51)–(18)]	(18)C		10^{-x}: $x = 0(.025)1$; 10D
[(23)]	(23)C, (24)		10^x: $x = 0(.00001)1$; 10D
		7.0	Trigonometric functions. Also see **24.2** and **28**.
	(4)	7.1	$\sin x$, $\cos x$: $x = 0(.001)6.283$; 5D
[(18)]	(18)C		$\sin x$: $x = 0(.001)1.571$; 7D
[(18)]	(18)C		$\sin x$: $x = 0°$(opt.int.)90°; 8D; 176 cards
[(23)–(11)]	(11)C		$\sin x$, $\cos x$: $x = 0(.1)20$; 8D
[(23)–(11)]	(11)C		$\sin x$, $\cos x$: $x = 0(.001)1.599$; 8D
[(26)]	(26)		$\sin x$: $x = 0(.01)99.99$, Δ; 8D
[—(3)]	(16), (23), (24)C		$\sin x$, $\cos x$; Δ, δ^2: $x = 0(.01)27$; 8D
	(4)		$\sin x$, $\cos x$, $\tan x$: $x = 0.0001(.0001)1.5709$; 9D for $\sin x$, $\cos x$; 8S (or less) for $\tan x$
[(23)–(18)]	(18)C		$\sin x$: $x = 0(.001)1.999$; 9D
[(23)–(18)]	(18)C		$\sin x$: $x = 0(.001)1.571$; 9D
[(66), (24)–(24)]	(24)C		$\sin x$, $\cos x$, Δ, δ^2: $x = 0(.0001)1$; 11D
[(23)–(12)]	(12)C, (24)	7.12	$\tan x$, $\cot x$: $x = 0(.0001)2$; mostly 8S on same card with $\tanh x$, $\coth x$

Source	*Available at*		*Description of Tables*
[(18)]	(10), (13), (18)C	7.2	$y = \tan f$: $f = 89°.942(.001)90°$; 3D *Note:* This is used by (9) as a critical table for $f = \arctan x$ in the region close to $f = 90°$.
[—(3)]	(24)C		$\sin x$, $\cos x$, $\tan x$, $\cot x$, Δ: $x = 0(°.1)360°$; 5D
[(68)–(27)]	(27)		$\sin x$, $\cos x$, $\tan x$, $\cot x$: $x = 0(°.001)90°$; 4S – 7S
[(69)–(21)]	(21)		$\sin x$, $\cos x$, $\tan x$: $x = 0(°.01)44°.99$; 5D
[(11)]	(11)C		$\tan x$: $x = 0(°.1)90°$
[(68)–(11)]	(11)C, (24)		$\sin x$, $\cos x$, $\sin kx$, δ^2; $k = 1(1)9$; $x = 0(°.1)360°$; 7D
[(68)–(17), (22)]	(10), (13), (16), (17)C, (18), (22)C, (23)C, (24)C		$\sin x$, $\cos x$, $\tan x$, Δ: $x = 0(°.01)90°$; 7S or 7D [range extended to 180° at (18); argument also given in deg., min. and sec. at (22)]
[(68)–(21)]	(21)		$\sin x$: $x = 0(°.001)89°.999$; 7D
[(47)–(22)]	(22)C		$\sin x$, $\cos x$, Δ: $x = 0(°.01)90°$; 8D
[(68)–(26)]	(26)		$\sin x$, $\tan x$, Δ: $x = 0(°.01)89°.99$; 8D
[(23)]	(23)C		$\sec x$, $\csc x$: $x = 0(°.01)90°$; 9S
[(23)–(24)]	(24)		$\sin x$, $\cos x$: $x = 0(1°)360°$; 15D
[(23)]	(24)C		$\sin x$, Δ: $x = 0°(°.01)90°$; 15D
[(23)]	(23)C		$\sin x$, $\cos x$, δ^2: $x = 0(°.01)90°$; 15D
[(23)]	(23)C, (24)C		$\sin x$, $\cos x$, $\csc x$, $\sec x$: $x = 0(°.01)45°$; 15D in $\sin x$ and $\cos x$; 8D in $\csc x$ and $\sec x$
[(27)]	(27)		$\sin x$, $\cos x$: $x = 0(°.01)90°$; 17S. Computed on IBM Relay calculator.
		7.3	Argument in grades or units of complete circle; $\frac{1}{2}\pi = 100^g$; $2\pi = 1^c$
[(51)–(14)]	(14)C		$\sin x$, $\cos x$; $0.25 - x$, Δ: $x = 0(0^c.0001)0.125$; 4D; 5D; and 6D
[(81)–(1)]	(1), (4)		$\sin x$, $\cos x$: $x = 0(^c.001)1$; 5D First difference given at (1).
[(18)]	(18)C		$\sin x$, $\cos x$, Δ^3: $x = 0(^c.01)1$; 8D (third order interpolation)
[(51)–(14)]	(14)C, (24)		$\sin x$, $\cos x$, Δ, δ^2; $0.25 - x$: $x = 0(^c.00001)0.125$; 15D Rounding error in last place may occasionally be greater than $\frac{1}{2}$ unit. At(14) C, Δ^2 is given in place of δ^2.

Source	*Available at*		*Description of Tables*
(26)	(26)	7.31	$\sin x$, $\cos x$: $\pm x = 0.001(^{g}.001)99.999$; 8D (First differences for functions at intervals of $^{g}.01$)
[(47)–(18)]	(18)C		$\sin x$, $\cos x$: $x = 0(1^{g})50$; 20D
		7.35	Trigonometric functions; argument in mils
[(26)–(21)]	(21)		$\sin x$, $\cos x$, $\tan x$, $\sec x$, $\csc x$, $\cot x$, $\frac{1}{2}\tan x$: $x = 0(.1)800$ mils; 5D (with extension to 1600 mils) $\frac{1}{2}\cot x$, $x = 50.9(.1)800$ mils; 5D
		7.4	Trigonometric functions; argument in degrees, minutes, and seconds
[—(3)]	(24)C		$\sin x$, $\cos x$, Δ, δ^2: $x = 0°(10')360°$; 5D
[(47), (28)–(28)]	(28)		$\cos x$, Δ per sec.: $x = 0(1')180°$; 5D, 6D
[(47), (28)–(28)]	(28)		$1 - \cos x$: $x = 90°(1')180°$; 5D
[—(3)]	(24)C		$\tan x$, $\cot x$; Δ, δ^2: $x = 0°(10')360°$; 5S
[—(3)]	(24)C		$\sec x$, $\csc x$; Δ, δ^2: $x = 0°(10')360°$; 5S
[—(3)]	(24)C		$\sin x$, $\cos x$, $\tan x$, $\cot x$, $\sec x$, $\csc x$: $x = 0°(10')360°$; 5D or 5S
[(45)–(4)]	(4)		$\sin x$, $\cos x$, $\tan x$, $\cot x$: $x = 0°1'(1')89°59'$; 5D or 5S
[(47), (28)–(28)]	(28)		$\tan^2 x$: $x = 0(1')88°$; 6D
[(80)–(17)]	(17)		$\tan x$: $x = 0(10'')30°$; 7D
[(23)]	(23)C		$\cos x$, $\Delta/60$: $x = 0°(1')90°$; 8D
[(47), (28)–(28)]	(28)		$\tan x$, $\cot x$, Δ per sec: $x = 0(1')90°$; 8D
[(47), (28)–(28)]	(28)		$\sin nx$, $n = 1, 2, 4, 6, 8$, Δ per sec: $x = 0(1')90°$; 12D for $n = 1$; 10D to 7D for $n \geq 2$. Also other sets to 4D, 5D, and 6D for some values of n.
[(47), (28)–(28)]	(28)		$\sin^2 x$, $\cos^2 x$: $x = 0(1')90°$; 10D
[(47)–(28)]	(28)		$\sin x$, $\cos x$, $\tan x$, $\cot x$, Δ per sec.: $x = 0(1')90°$; 15D
[(47)–(70)]	(23)C, (26), (28)		$\sec x$, $\csc x$: $x = 0°(1')90°$; 10D
[(23)]	(23)C	7.44	$\sin y$, $\cos y$, $\Delta/60$; $y = (b/a)\tan x$, where a and b are the major and minor axes of the ellipse on Clarke's spheroid (1866): $x = 0(1')90°$; 8D

Source	*Available at*		*Description of Tables*
[(11)]	(11)C	7.63	$\frac{\sin x}{x}, \frac{\cos x}{x}; \frac{d^n}{dx^n}\left(\frac{\sin x}{x}\right)$: $x = 0(.1)20$; 8D
		7.8	Area and circumference of circle $A = \frac{1}{4}\pi D^2$; $C = \pi D$: A, $D = 0(.0001)0.9999$: 8D in A; 7D in D
[(56)–(21)]	(21)C	7.9	(a) $\xi(\theta) = \int_0^\theta \sec^3 t\, dt$; $1 - \beta = \xi/b$: $\theta = 0(1')87°$; 4D to 7D
			(b) $\sin\theta(\xi)$, $\cos\theta(\xi)$, where $\theta(\xi)$ is the inverse of $\xi(\theta)$: $\xi = 0(.01)2(.05)50$; 8D or 9D
			(c) $X(\beta, b) = \int_1^\beta \cos\{\theta[b(1-t)]\}\frac{dt}{t}$
			(d) $Y(\beta, b) = \int_1^\beta \sin\{\theta[b(1-t)]\}\frac{dt}{t}$
			(e) $T(\beta, b) = \int_1^\beta \cos\{\theta[b(1-t)]\}t^{-\frac{1}{2}}dt$
			X, Y, T: $\beta = 0.02(.02)3$; $b = 0.1(.1)2$; 8D
[(24)]	(24)C		$f(\cos x, \alpha, k) = k\left\{\frac{\cos x}{\alpha} + \frac{1 - \frac{\cos x}{\alpha}}{(1 + \alpha^2 - 2\alpha\cos x)^{\frac{1}{2}}}\right\}$: $\cos x = -1(.01)1$; 5S or 6S
			$\alpha = 0.015, 0.016, 0.017, 0.018$, with $k = 12045.4$
			$\alpha = 0.0000425$, with $k = 0.02480289$
		8.5	Tables with arguments in time. See 28.5
		9	Inverse circular functions
[(18)]	(18)C	9.10	$\arcsin x$: $x = 0.005(.01)0.995$; 7D
[(23)–(7)]	(7)C, (24)C		$\arcsin x$, $\arcsin(x-h)$, $\arcsin(x-2h)$, δ_0^2, δ_{-1}^2 $x = 0(.0001)0.989(.00001)0.99999$; 12D
[(18)]	(18)C		$x_1 = \arcsin f$; $f = 0(.01)1(-.01)-1(.01)0$; 2D
			$x_2 = \arccos f$; $f = 1(-.01)-1(.01)1$; 2D
			$x_1, x_2 = 2\pi\lambda$, $0 \le \lambda \le 1$; 2D in λ (critical table of $\sin x$, $\cos x$)
[(23)–(18)]	(18)C		$2\pi\lambda = \arcsin x$: $x = 0.005(.01)0.995$; 5D in λ

Source	*Available at*		*Description of Tables*
[(68)–(14)]	(14)C		y = arccos x and arccos $-x$ (y in degrees): x = 0.99899(.00001)1; accuracy to within 0°.01
[(68)–(14)]	(14)C		y = arccos x (y in degrees): x = 0(opt.int.)0.99899; accuracy to within 0°.01; 95 cards
[(18)]	(18)C	9.12	arctan x: x = 0(opt.int.)100; 6D; 110 cards arctan given in radians
[(23)–(11)]	(11)C		arctan x: x = 0(.001)7; 6D
[(26)]	(26)		arctan x: x = 0(.001)1; 10D
[(23)–(24)]	(24)C		arctan x, ∇, δ^2: x = 0(.001)7(.01)50(.1)300(1)2000(10)10,000; 12D. [∇ is the "backward" first difference.]
[(23)–(10)]	(10)		arctan x: x = 0(.001)1(1)10(10)100(100)1000; 12D
[(23)–(27)]	(27)		arctan x: x = 0(1)2000(10)9000; 12D
[(42)–(24)]	(23)C, (24)C		arctan m/n, arccot m/n, $m^2 + n^2$: m = 1(1)n; n = 1(1)100; 12D
[(23)–(24)]	(16), (24)C		arctan x: x = 1(1)2000; 15D
[(24)]	(24)C		n arctan 1 = $n\frac{\pi}{4}$: n = 0(1)100; 20D
[(21)]	(21)	9.15	y = arctan x: x = 0(.001)3.75(.01)18.25; (y in degrees); accuracy to 0°.01. [Extension for $x > 18.25$ by special table.]
[(18)]	(10), (13), (18)C		y = arctan x: x = 0(opt.int.)1000; (y in degrees) accuracy 0°.001; 350 cards
[(23)–(22)]	(22)C		y = arctan x, Δ: x = 0(.001)1000; (y in degrees); 6D
		9.7	General spherical triangle (see **28.4**)
		10	Exponential and hyperbolic functions
[(23)–(13)]	(13)C	10.0	e^x, e^{-x}: x = 0(.00001)0.00099; 7D or 8D
[(23)–(13)]	(13)C		e^x, e^{-x}: x = 0.001(.001)0.999; 7D or 8D
[(23)–(13)]	(13)C		e^x, e^{-x}: x = 1(1)15; 8S
[(23)–(4)]	(4)		e^x, e^{-x}: x = 0(.0001)1; 10D for e^x; 9D for e^{-x}
[(45)–(18)]	(18)		e^{-x}, Δ: x = 0(.01)3(.1)10; 6D
[(23)]	(24)C		e^{-x}: x = 0(.1)10; 10D
	(16), (24)		e^{-x}: x = 0(.001)2.499; 10D

Source	*Available at*		*Description of Tables*
[(65)–(24)]	(16), (24)C		e^x, e^{-x}; Δ, δ^2, Δ^2: $x = 2.5(.001)10$; 10D
[(79)–(4)]	(4)		e^x, e^{-x}: $x = 1(1)100$; 10S
[(23)–(21)]	(21)		e^x: $x = 2.5(.001)5(.01)10$; 15D for $x \leq 5$; 12D for $x > 5$
[(27)]	(27)		e^x: $x = -100(.01)100$; 15S. Computed on IBM Relay Calculator.
[(23)–(14)]	(14)C		e^x, e^{-x}: $x = 0(10^{-6})0.0001$; 18D
[(23)–(24)]	(24)		e^{-x}: $x = 0(.01)2.49$; 18D
[(23)–(14)]	(14)C, (21), (24)		e^x, Δ, Δ^2, Δ^3, Δ^4: $x = -2.5(.0001)2.5$; 18D for $x < 1$; 15D for $x \geq 1$
[(23)–(24)]	(16), (24)		e^x: $x = 0(.001)2.499$; 18D for $x < 1$; 15D for $x \geq 1$
[(27)–(26)]	(23), (26)		e^x, e^{-x}: $x = 0(.01)100$; 14S and 18S
[(79)–(14)]	(14)C		e^x, e^{-x}: $x = 1(1)100$; 19S
[(14)]	(14)C, (24)	10.33	$(1 - e^{-x})/x$: $0 \leq x \leq 13.0369$; 6D; (pseudo-opt. int.); 792 cards
[(26)]	(26)	10.4	$\sinh x$, $\cosh x$, Δ, δ^2: $x = 0(.0001)0.01(.001)1(.01)99.99$; 8D
[(23)–(4)]	(4)		$\sinh x$, $\cosh x$: $x = 0(.0001)1$; 9D
[(26)]	(26)		$\sinh x$, $\cosh x$: $x = 0(.01)16.11$; 16D
[(23)–(12)]	(12)C, (24)		$\tanh x$, $\coth x$: $x = 0(.0001)2$; mostly 8S
	(26)		$\tanh x$, Δ: $x = 0(.01)9.9$; 8D
[(18)]	(18)C		$\tanh x$: $x = 0(.005)6$; 9D
		11	Natural logarithms
[(23)–(21)]	(21), (23)	11.1	$\log_e x$: $x = 1(1)100{,}000$; 16D
	(16)	11.2	$\log_e x$: $x = 2(.01)6.6$; 7D
[(24)]	(24)		$\log_e x$, Δ: $x = 1(.001)9.999$; 7D
[(23)–(4)]	(4)		$\log_e x$: $x = 0(.0001)1$; 16D
[(23)–(21)]	(21)		$\log_e x$: $x = 0.0001(.0001)10$; 16D
[(18)]	(18)C		$\log_e x$, (radix table): $0.001 \leq x \leq 9.9999$; 5D; 76 cards
[(18)]	(18)C	11.26	$\log_e(-\log_e x)$: $x = 0(.0001)0.001$-$(.001)0.999(.0001)0.9999$; 4D
[(18)]	(18)C, (23)		$\log_e(-\log_e x)$; Δ, Δ^2: $x = 0(.00001)$-$0.006(.0001)0.08(.001)0.84(.0001)$-$0.984(.00001)0.99995$; 5D
		11.6	Inverse hyperbolic functions: $\operatorname{arctanh} x = \frac{1}{2}\log_e[(1+x)/(1-x)]$; $\operatorname{arc\,sinh} x = \log_e(x + \sqrt{1+x^2})$
[(23)–(18)]	(16), (18)C		$\operatorname{arctanh} x$: $x = 0(.002)0.5$; 6D (Δ also on cards at 18)

Source	*Available at*		*Description of Tables*
[(50), (14)–(14)]	(14)C		arcsinh x, arc tanh x: $x = 0.00001(.00001)0.001$; 20D and $x = 0.001(.0001)0.1(.001)0.999$; 7D to 10D
[(50), (14)–(14)]	(14)C		arcsinh x, arccosh x $x = 1(.001)3(.01)10(.1)20(1)50$; 7D and 8D
[(10)]	(10)	11.7	Logarithms of complex numbers $f = \log_e z$, $z = x + iy$: $x, y = -4(.02)4$; 5D
		11.8	Addition logarithms
[(14)]	(14)C		$\log_e(1 + e^x)$: $x = 0$(opt.int.)13; 5D; 270 cards
		13	Exponential integrals for real and complex arguments; sine and cosine integrals $E_n(x) = \lim_{c \to +\infty} \left[x^{n-1} \int_x^{x+c} u^{-n} e^{-u} du \right]$; c real
[(23), (14)–(14)]	(14)C	13.2	$E_1(x)$: $x = 0.01(.01)10(.1)90$; about 9S
[(23)–(16)]	(16)C	13.4	Si(x), Δ, δ^2: $x = 0(.01)99.99$; 10D
[(23)–(16)]	(16)C	13.5	Ci(x), Δ, δ^2: $x = 0(.01)99.99$; 10D
[(23), (2)–(2)]	(2)	13.52	Modified cosine integral: $-\text{Ci}(x) + \gamma + \log_e x$ $x = 0(.001)10(.01)49.99$; 6D
[(23), (24)–(24)]	(24)C	13.8	$E_2(x)$: $x = 0(.001)2$; 7D $x = 2(.001)10$; 5S or 6S; $x = 10(.001)16$; 5S to 8S
		13.9	Exponential integrals for complex arguments; $z = x + iy$
[(23)]	(23)C		$E_1(z)$: $x = 0(.02)4$; $y = 0(.02)3(.05)10$; 6D $E_1(z) + \log_e z$: $x, y = 0(.02)1$; 6D $e^z E_1(z)$: $x = 3(.1)10$; $y = 0(.05)10$; 6D
[(23)]	(23)		$e^z E_1(z)$: $-x = 0.5(.5)4.5$; $y = 0(.1)4(.5)10$; 6D $-x = 5(.5)10$; $y = 0(.5)10$; 6D $x = -10(1)10$; $y = 10(1)20$; 6D $\pm x = 11(1)20$; $y = 0(1)20$; 6D [Branch cut on negative imaginary axis]

Source	*Available at*		*Description of Tables*
			Available at (23); only partially punched at time of publication of this guide.
[(24)]	(24)C		$E_1(z)$: $-x = 0(.1)3.1$; $y = 0(.1)3.1$; 10D
			$E_1(z) + \log z$: $-x = 0(.1)1$; $y = 0(.1)1$; 10D
		14	Gamma function
[(63), (21)–(21)]	(21)	14.2	$100 + \log_{10} \Gamma(x)$: $x = 1(.01)16.99$; 7D
[(23)]	(23)C	14.6	$\text{Log}_e \Gamma(z)$, $z = x + iy$: $x = 9(.1)10$; $y = 0(.1)10$; 14D or 15D
			[Extension of range to $x = 0(.1)10$ is in progress at time of publication of this guide.]
		15	Probability functions; Hermite polynomials; moments
			$z(x) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}x^2)$;
			$\alpha(x) = \int_{-x}^{x} z(t)dt$
[(23)–(14)]	(14)C	15.1	$z(x)$, $\frac{1}{2}\alpha(x)$: $x = 0$(opt.int.)4.751; 5D; 886 cards
			Prepared for linear interpolation; fixed interval.
[(23)–(14)]	(4), (14)C, (24)		$z(x)$, $\frac{1}{2}\alpha(x)$, Δ, δ^2: $x = 0(.001)7.8$; 8D
			At (14)C without differences.
[(23)–(14)]	(14)C		$z(x)$, $\alpha(x)$: $x = 0(.01)7$; about 10D
			[with auxiliary functions, prepared for third-order interpolation].
[(23)–(14)]	(4), (14)C, (23), (24) (26)		$z(x)$, $\alpha(x)$, Δ: $x = 0(.001)7.8$; 15D
			At (14)C and (26) without differences.
	(24)		$e^{-\frac{1}{2}x^2}$: $x = 0(.05)6.25$; 8D
[(4)]	(4)	15.281	$A = Be^{-x^2}$; $B = \int_0^x \exp t^2 dt$:
			$x = 0.1(.01)9.99$; 10D in A; 8S, 9S, or 10S in B
		15.411	Inverse tables
[(62)–(14)]	(14)C		$x(p)$, where $p = \frac{1}{2}[1 + \alpha(x)]$, and $z(x)$ $p = 0.5(.0001)0.9999$; 8D
[(23)–(14)]	(14)C, (24)	15.521	$\frac{2}{\sqrt{\pi}} \frac{d^n}{dx^n}(e^{-x^2})$: $x = 0(.1)5.9$;
			$n = 1(1)15$; $(24 - n)$D

Source	*Available at*		*Description of Tables*
[(23)–(14)]	(14)C, (24)	15.523	$\sqrt{2/\pi}\,\frac{d^n}{dx^n}(e^{-\frac{1}{2}x^2})$: $x = 0(.1)8.4$; $n = 1(1)16$; $(24 - n)$D
[(14)]	(14)C	15.6	$\frac{d^n}{dx^n}z(x)$ and $\frac{1}{z(x)}\frac{d^n}{dx^n}z(x)$ (Hermite polynomials) $n = 1(1)10$; $x = 0(.01)12$; about 6S
[(14)]	(14)C		Hermite polynomials: $n = 1(1)10$, $x = 0(.01)12$; exact
[(26)]	(26)	15.7	$m_i(x) = \frac{1}{(i-1)!!\sqrt{2\pi}}\int_0^x t^i \times \exp(-\frac{1}{2}t^2)dt$ [see (40)]
			$M_i = m_i^{\frac{1}{i}}\left(\frac{1}{M}\right)$: 5D; $\exp(M_1)$; 6D; $\exp(M_i - M_1)$; 2D
			$[\exp(-M_i) - \exp(-M_{i+1})]/\exp(-M_i)$; 6D
			$[M_i\exp(-M_i) - M_{i+1}\exp(-M_{i+1})]/[\exp(-M_i) - \exp(-M_{i+1})]$; 6D
			$1/M = 0.24(.01)0.99$; $i = 1(1)8$
[(14)]	(14)C		$F_m(x) = \frac{1}{2}\left[\int_0^x (\frac{1}{2}t)^{\frac{1}{2}m-1}e^{-\frac{1}{2}t}dt\right]/\Gamma(\frac{1}{2}m)$ $m = 2(1)10$; $x = 0$(opt.int.)44; 5D; 7637 cards
		15.8	Probability integrals relating to complex arguments
[(14)]	(14)C, (24)		$\phi_2(x, y) + i\phi_1(x, y) = \int_0^x \exp(ity)z(t)dt$; 5D
			$\phi_3(x, y) = \int_x^\infty [\sin ty]z(t)dt$; 5D
			$x = 0(.1)4.5$, $y = 0(.1)0.9$; $x = 0(.05)2(.1)4.5$, $y = 1(.1)1.9$; $x = 0(.05)3(.1)4.5$, $y = 2(.1)2.9$; $x = 0(.02)1.5(.05)3(.1)4.5$, $y = 3(.1)3.9$; $x = 0(.02)2(.05)3.5(.1)4.5$, $y = 4(.1)4.9$; $x = 0(.02)2.5(.05)3.5(.1)4.5$, $y = 5(.1)6$. Altogether 6216 cards.
[(14), (24)]	(14)C	15.9	$q(R, x) = \int_R^\infty te^{-\frac{1}{2}(t^2+x^2)}I_0(tx)dt$ (Offset circle probabilities) $R = 0.1(.1)20$; $x = 0(.05)1$; 6D
		17–20	Bessel functions

Source	*Available at*		*Description of Tables*
[(5)–(16)]	(16)C	17.1	$J_0(x)$, $J_1(x)$, Δ: $x = 0(.1)25$; 6D
[(5)–(16)]	(16)C		$J_0(x)$, $J_1(x)$, $\Delta J_0(x)$: $x = 0(.1)99.9$; 6D
[(23)]	(14)C, (24)		$J_0(x)$, $J_1(x)$: $x = 0(.01)10$; 10D First four advancing differences given along with functions at (14)C.
[(66)–(14)]	(14)C, (24)		$J_0(x)$, $J_1(x)$: $x = 10(.01)25$; 10D, Δ, Δ^2, Δ^3, Δ^4
[(66)–(11)]	(11)C		$J_0(x)$, $J_1(x)$: $x = 0(.01)25$; 10D
[(5)–(16)]	(16)C		$J_2(x)$, $J_3(x)$: $x = 0(.1)25$; 8D
[(5)]	(5)C		$J_n(x)$: $n = 0(1)15$; $x = 0(.001)25(.01)99.99$; 18D for $n \leq 3$; 10D for $n \geq 4$ $n = 16(1)100$; $x = 0(.01)99.99$; 10D; $J_n(100)$: $n = 0(1)100$; 10D *Note:* At the time of preparation of this guide, values for $n = 79(1)100$ and $J_n(100)$ were not yet available for distribution.
[(23)–(16)]	(16)C	17.2	$\sqrt{\frac{\pi}{2x}}\, J_{n+\frac{1}{2}}(x)$: $n = 1, 2$; $x = 0(.1)16$; 8D
[(66), (16)–(16)]	(16)	17.3	$Y_n(x)$, Δ: $n = 0(1)3$; $x = 0.01(.01)1$; 6D [$Y_0(x)$ and $Y_1(x)$ checked]
[(66)–(11)]	(11)C		$Y_0(x)$: $x = 0(.01)25$; 8D
[(66)–(14)]	(14)C, (24)		$Y_0(x)$, $Y_1(x)$: $x = 10(.01)25$; 8D
[(66), (16)–(16)]	(16)		$Y_n(x)$, Δ: $n = 0(1)3$; $x = 0.5(.1)25$; 8D [$Y_0(x)$ and $Y_1(x)$ checked]
[(23)]	(23)C, (24)C, (26)		$Y_0(x)$, $Y_1(x)$: $x = 0(.01)10$; 10D
[(53)–(24)]	(24)		$Y_n(x)$: $x = 0(.1)10$; $n = 1(1)21$; 10S
[(66)–(24)]	(24)C	18.1	$I_0(x)$ and $I_1(x)$, δ^2: $x = 0(.001)5$; 8D
[(23)–(14)]	(14)C, (23)C		$I_0(x)$, $I_1(x)$: $x = 0(.01)10$; 10D
[(66), (16)–(16)]	(16)		$I_n(x)$ or $e^{-x}I_n(x)$: $n = 0(1)3$; $x = 0(.1)20$; 8D [$e^{-x}I_n(x)$ for $x > 5$; entries for $n = 0, 1$ checked]
[(23)–(14)]	(14)C	18.2	$I_{\frac{1}{2}}(x)$ and $I_{-\frac{1}{2}}(x)$; $x = 0.01(.01)25$; 10S
[(66)–(24)]	(24)C	18.3	$K_0(x)$, $K_1(x)\delta^2$; also auxiliary functions for interpolating near the origin: $x = 0(.01)5$; 7D to 10D

Source	*Available at*		*Description of Tables*
[(66), (16)–(16)]	(16)		$K_n(x)$ or $e^x K_n(x)$: $n = 0(1)3$; $x = 0.5(.1)20$; 8D [$e^x K_n(x)$ for $x > 5$; entries for $n = 0$ and 1 checked]
[(48)–(21)]	(21)	18.4	$e^{-x}I_0(x)$, $e^{-x}I_1(x)$, $e^x K_0$, $e^x K_1$, e^x: $x = 0(.02)16$; 7D
[(23), (14)–(14)]	(14)C		$e^{-x}I_0(x)$, $e^{-x}I_1(x)$: $0 \leq x \leq 733.4104599$; 8D or better; 1996 cards [Quasi-optimum interval, arranged for quadratic interpolation, with modified first and second divided differences.]
[(14)]	(24)C		e^{-x}, $e^{-x}I_0(x)$, $e^{-x}I_1(x)$; Δ, δ^2: $x = 0(.01)2.49$; 18D for e^{-x}; 8D for other functions
[(14)]	(24)		$e^{-x}I_0(x)$: $x = 0(.01)36$; 8D
[(14)]	(24)		$e^{-x}I_1(x)$: $x = 0(.01)72$; 8D
[(66)–(24)]	(24)		$e^{-x}I_0(x)$, $e^{-x}I_1(x)$, $e^x K_0(x)$, $e^x K_1(x)$; δ^2: $x = 5(.01)10(.1)20$; 8D
[(66)–(24)]	(24)		$e^{-x}I_0(x)$: $x = 2.5(.01)5$; 9D
[(24)]	(24)		$e^{-x}I_1(x)$: $x = 20(.5)120(1)625$; 8D to 10D
[(14)]	(24)		$e^{-x}I_0(x)$: $x = 20(1)48$; $37.5(2.5)92.5$; $75(5)175$; $160(10)380$, $325(25)800$; 10D
[(14)]	(24)		$e^{-x}I_1(x)$: $x = 16(1)48$, $37.5(2.5)92.5$; $70(5)170$; $130(10)430$, $325(25)700$; 8D, 9D or 10D
		19.0	Bessel functions for complex arguments
[(23)]	(23)C, (24), (26)		$Y_0(z)$ and $Y_1(z)$: $z = \rho \exp(i\phi)$; $\rho = 0(.01)10$; $\phi = 0(5°)90°$; 10D
[(23)]	(23)C, (24)C		$Y_0(ix)$, $Y_1(ix)$: $x = 0(.01)10$; 10D
		20.0	Bessel-Clifford functions
[(23)]	(23)C, (24)		$x^{-\frac{1}{2}n}J_n(2\sqrt{x})$, and $x^{-\frac{1}{2}n}Y_n(2\sqrt{x})$: $n = 0, 1$; $x = 0(.02)1.5(.05)3(.1)13(.2)45(.5)115(1)410$; 8D or 9D
[(24)]	(24)		$I_0(2\sqrt{x})$, $[I_1(2\sqrt{x})]/\sqrt{x}$; δ^2; $K_0(2\sqrt{x})$, $[K_1(2\sqrt{x})]/\sqrt{x}$: $x = 0(.02)1.5(.05)6.2$
[(24)]	(24)		$e^{-2\sqrt{x}}I_0(2\sqrt{x})$, $[e^{-2\sqrt{x}}I_1(2\sqrt{x})]/\sqrt{x}$, $e^{2\sqrt{x}}K_0(2\sqrt{x})$, $[e^{2\sqrt{x}}K_1(2\sqrt{x})]/\sqrt{x}$, δ^2: $x = 6.2(.1)13(.2)36(.5)115(1)160(5)410$; 7D to 9D

Source	*Available at*		*Description of Tables*
[(49)–(24)]	(24)C	20.8	$\int_0^x J_0(\lambda u) \exp iu\,du = J_c(\lambda, x) + iJ_s(\lambda, x)$ $\int_0^x Y_0(\lambda u) \exp(iu)du = N_c(\lambda, x) + iN_s(\lambda, x)$ Auxiliary functions: $C_c(\lambda, x) = N_c(\lambda, x) - \frac{2}{\pi}[\ln(\lambda x)]J_c(\lambda, x)$ $C_s(\lambda, x) = N_s(\lambda, x) - \frac{2}{\pi}[\ln(\lambda x)]J_s(\lambda, x)$ $J_c(\lambda, x)$, $J_s(\lambda, x)$, $N_c(\lambda, x)$, $N_s(\lambda, x)$: $x = 0(.02)2(.1)5$; $\lambda = 0.1(.1)1$; 6D Auxiliary functions: $x = 0(.02)x_c$; $x_c < 1.6$; 6D
		22	Riemann Zeta function; Mathieu functions
[(78)–(16)]	(16)C, (24)	22.1	$(z - 1)\zeta(z)$; $z = x + iy$; $x = 0(.05)2$; $y = 0(.05)4$; 6D [computed on the EDSAC]
[(61)–(24)]	(24)	22.2	(*a*) Characteristic values, $be_r(s)$ and $bo_r(s)$, associated with even and odd periodic solutions, respectively (period π or 2π), of Mathieu's equation $y'' + (b - s^2 \cos x)y = 0$ $b = be_r(s)$, $bo_r(s)$, δ^2; $0 \leq s \leq 100$; $r \leq 15$; 8D (*b*) Trigonometric coefficients $De_k^{(r)}(s)$ and $Do_k^{(r)}(s)$ associated with periodic solutions, same range of r and s, but at larger intervals; 9D or 9S (*c*) Joining factors, relating various solutions, δ^2; mostly 8S, same range of s and r. For precise definitions see (61).
[(23)]	(23)C, (24)		(*d*) Periodic solutions $Se_r(s, x)$ and $So_r(s, x)$; $0 \leq s \leq 100$, $r = 6(1)15$; $x = 0(1°)90°$; 7D
[(72)]	(24)		Solutions $g(t)$ and $h(t)$ of $y'' + \epsilon(1 + k \cos t)y = 0$; $g(0) = h'(0) = 1$; $h(0) = g'(0) = 0$ $\epsilon = 1(1)10$, $k = 0.1(.1)1$; $t = 0(.1)3.1$; $3.14(.0004)3.1428$

Source	*Available at*		*Description of Tables*
			$\epsilon = 30(10)90,200, 300, 400; k = 0.02;$ $\epsilon = 900, k = 0.01$
			$\epsilon = 5, k = 0.12(.02)(.24)$ and $1.1(.1)1.9$
			Functions tabulated: $g(t)$, $g'(t)$, $h(t)$, $h'(t)$; 3D to 5D
			[computed on the ENIAC]
		22.73	See **24.2**, Harmonic analysis
[(13)]	(13)C, (16), (18)C	**23.1**	4-Pt. Lagrangean interpolation coefficients: $x = 0(.0001)0.5$; 8D
			[7D at (18)]
			[from symmetry, this is equivalent to giving coefficients up to $x = 1$]
[(23)–(4)]	(4)C, (24)		6-Pt. Lagrangean interpolation coefficients: $x = 0.001(.001)0.999$; 10D
[(59)–(15)]	(15)C	**23.82**	Orthogonal polynomials for curve fitting
			$\xi_0'(x) = \lambda_0$; $\xi_1'(x) = \lambda_1(x - \bar{x})$; $\bar{x} = \frac{1}{2}(n + 1)$
			[Orthogonal with respect to summation from 1 to n.]
			$\xi_k'(x)$ and λk: $k = 1(1)5$; $n = 3(1)104$; exact
			See **23.82** of (40); *MTAC*, v. 1, p. 148–150, (59), and (58), p. 307–327, for uses of these polynomials.
[(13)]	(13)C	**24.2**	Harmonic analysis
			$A \sin 2\pi nx$, $A \cos 2\pi nx$
			$n = 1(1)30$; $x = 0(1)120$; $A = 1(1)10(10)100(100)900$
			5D for $A = 1$; 1D for others
[(20), (23)–(20)]	(20)		$A \sin 2\pi nx$, $A \cos 2\pi nx$
			$n = 0(1)30$; $x = 0(.002)0.25$; $\pm A = 1(1)5$; 10(10)50; 100(100)500; 3S–5S
			$n = 0(1)20$; $x = 0\left(\frac{1}{60}\right)\frac{15}{60}$; $\pm A = 1(1)5$; 10(10)50; 100, 200, 500; 1S–3S
			$n = 0(1)40$; $x = 0\left(\frac{1}{120}\right)\frac{15}{120}$; $\pm A = 1(1)33; \frac{100}{3}\ \frac{(100)}{3}\ \frac{1,000}{3}$; 1S–3S
			$A \sin 2\pi nx$: $n = 1(1)143$; $x = 0.0005(.0005)0.25$; 3S–5S

Source	*Available at*		*Description of Tables*
[(18)]	(18)C		$f_1 = \cos z$; $f_2 = \sin z$: $z = 0.01\,mn\,\pi/20$; $m = 1(1)600$; $n = 1(1)200$; 6D $f_3 = (\sin z)/z$; same range of z; 5D. Total cards: 40,200 *Note:* Same range of z, at double the interval in m and n for f_1 and f_2 and at four times the interval in m and double the interval in n *fw* f_2, to four decimals, on 7500 cards. Additional table of f_3, without f_1 and f_2, same range of z; 5D; 13,400 cards
[(18)]	(18)C		$f_1 = A\cos(2\pi mn/k)$; $f_2 = A\sin(2\pi mn/k)$: $m = 0, 1, 2$; $n = 0, 1, 2$; $k = 3$ $m = 0, 1, 2, 3, 4$; $n = 0, 1, 2, 3, 4$; $k = 5$ $A = 0(.000001)0.000999$ and $-0.499(.001)0.500$; 7D
[(24)]	(24)C		$\sin nx$, $\cos nx$: $n = 1(1)31$: $x = 0(1°)90°$; 15D
[(71)–(12)]	(12)C, (24)		$S(k, n) = \int_0^1 x^k \sin(n\pi x)dx$; $C(k, n) = \int_0^1 x^k \cos(n\pi x)dx$ $S(k, n)$ and $C(k, n)$; $k = 0(1)10$; $n = 1(1)100$; 10D
		25	Random numbers
[(14)]	(4), (14)C, (23), (24)	**25.1**	Digits of 0(1)9 drawn at random, with equal probability. 50 random digits per card; 20,000 cards
[(14)]	(24)		Random numbers, positive and negative; logs of random numbers, numbered serially 0(1)6999
[(14)]	(4), (14)C, (24)	**25.2**	Random Gaussian deviates, x from $N(0.1)$. Range: $-4.417 \leq x \leq 4.417$; x to 3D; x^2 to 6D 10 deviates per card, 10,000 cards
[(4)]	(4)	**25.3**	Cosines of random angles; 9D Cards 0(1)14,566; random angle β: $0 \leq \beta \leq 9.4247$
[(14)]	(14)C	**25.4**	Correlated Gaussian deviates from $N(0, 1)$ $x_{n+1}(\rho) = \rho x_n(\rho) + \sqrt{(1-\rho^2)}\xi_n$,

Source	*Available at*		*Description of Tables*
			where ξ_n is a random Gaussian deviate, ρ is the correlation coefficient, and $x_0(\rho)$ is an uncorrelated Gaussian deviate. $\rho = 0.6, 0.8, 0.9, 0.95, 0.97, 0.98, 0.99, 0.995$; $n = 0(1)9999$; 3D
	(6) see (54)	**26**	Tables relating to aerodynamics A wide range of tables on punched cards, relating to shock waves and other phenomena, for various Mach numbers; "Busemann" coefficients, altitude functions, etc., are available at (6). See (54).
		27	Tables relating to number theory
[(26)]	(26)	**27.1**	Powers of primes. 2^n: $n = 1(1)100$; exact; [see also **2.3**]
[(66)–(16)]	(16)C		Table of primes p: $p < 10{,}000$
[(16)]	(16)C	**27.2**	Powers of primes p for all primes less than 1580 p^k: $k = 1(1)5$; exact; also $1 + p^2 + p^4$; $\sum_{k=0}^{N} p^k$; exact, for $N = 2, 3, 4, 5$
[(21)]	(21)	**27.25**	Fermat's quotients and extraordinary primes. See description in *MTAC*, v. 5, p. 84–85, [(73), file 117(F)]. n, p, p^2, ϵ, f, r, R: $n = 2(1)2962$ [computed on the ENIAC]
	(24)	**27.3**	The largest prime factor of x: $x = 1(1)4200$
[(60)–(19)]	(73) Ref. 114	**27.4**	Sums of fifth powers of the divisors of n: $n = 1(1)5000$; exact [See (60)b]
[(60)–(19)]	(73) Ref. 101		Ranamujan's function $\tau(n)$; [See (60)a] (a) $\tau(n)$; $\sum_{n \le N} \lvert\tau(n)\rvert$; $\sum_{n \le N} \{\tau(n)\}^2$: $n = 1(1)2500$; $N = 10(10)2500$; exact (b) $\tau(p)$, p a prime: $1000 < p < 2500$; exact (c) $\lvert\tau(p)\rvert p^{-11/2}$; 6D
[(74)–(19)]	(19)	**27.5**	Stencils for the solution of systems of linear congruences modulo 2; 1024 cards

Source	*Available at*		*Description of Tables*
[(74)–(19)]	(73) Ref. 125	27.6	Table of integer solutions of $\lvert y^3 - x^3 \rvert < x$ Range: $x < (10^6)/9$ or $y < (10^9)/27$
		28	Tables relating to astronomy [also see 7.44]
[(18)]	(18)C	28.1	Calendar date to Julian date 1853 (1 mo.) 1950; 92 cards
[(52)–(18)]	(18)C	28.2	[Kepler's equation] $E = M - e \sin E$: $e = 0(.01)0.38$; $M = 0(1^\circ)360^\circ$; accuracy: $0^\circ.01$; 14,079 cards
[(55)–(24)]	(24)C	28.3	Tables for rocket and comet orbits (Kepler's equation) [See (55) under Sources, for definitions] (a) Elliptic orbits: $C_n(U)$, $S_n(U)$, $X_n(U)$, Δ, δ^2: $U = 10^{-3n}u$; $n = 1, 2, 3, 4$, and $n \geq 5$: $0.15 \leq u \leq 150$ $n = 0$: $0.15 \leq u \leq 3.15$ (b) Hyperbolic orbits: $C_h(U)$, $S_h(U)$, $X_h(U)$, Δ, δ^2: $n = 1, 2, 3, 4$ and $n = 5$: $0.15 \leq u \leq 150$ $n = 0$: $0.15 \leq u \leq 300$ mostly 7S and 8S
[(76), (77)–(22)]	(22)C	28.4	General spherical triangles Arguments: sides a, b; angle C Functions tabulated: $(90-c)$(degrees and min.) and B (degrees) Range: $a = 0(1^\circ)89^\circ$; $b = 61^\circ(1^\circ)90^\circ$ to 119° $C = 0(1^\circ)$ upper limit The upper limits of b and C are such that c covers the range 0 to 90°. Δc along b.
		28.5	Tables with arguments in time [$1^s = 15''$; $1^m = 15'$]
[(17)]	(17)C		$\tan x$, $\sec x$: $x = 0(1^s)20^m$; 7D
[(17)]	(17)C		$\tan x$, $\sec x$: $x = 0(^s1)20^m$; 8D
[(22)]	(22)	28.6	Navigation Table H. O. 218 This table tabulates altitudes (-5° to $+90^\circ$) and azimuth (0 to 360°) as functions of declination (-30° to $+30^\circ$), local hour angle (0 to 360°),

Source	*Available at*		*Description of Tables*
			and latitude ($-90°$ to $+90°$). Complete information about this and similar punched card tables is available at (22).
	(30)	**29**	Actuarial tables Tables of "CSO Monetary Values," comprising a wide range of tables relating to annuities, etc., at various interest rates, have been deposited with (30); write for more complete information. Additional tables based upon an interest rate of $2\frac{1}{2}\%$ and the use of continuous functions, prepared by the John Hancock Mutual Life Insurance Company, can be obtained from (30).
		30	Tables relating to map projections and geodesy
[(23)–(28)]	(28)	**30.1**	Meridional arc; $x = 0(1')90°$; length to 0.001 meters
[(28)]	(28)	**30.2**	Tables for the conversion of latitude and longitude to grid coordinates and vice versa, on the following spheroids: Hayford, Clarke (1886) and (1880), Bessel: all for lat. 0–80 Everest: lat. 0–45 All at 1′ intersections
	(28)	**30.3**	Tables for polar stereographic grid coordinates, north and south polar areas, lat. $79°30' - 90°$.
	(28)	**30.4**	Tables for the universal transverse Mercator projection (for computation of plane coordinates to 1 cm. accuracy)
			Also see 7.44. For other punched cards relating to map projections write to (28).

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Formulas for Finding the Argument for Which a Function Has a Given Derivative

The problem of finding where a function $f(x)$ that is tabulated at equal intervals h has a given derivative (i.e. given $f'(x) \equiv f_x' \equiv f'(x_0 + ph)$, to find p) arises very frequently in numerical computation, especially in the location of a maximum or minimum point when $f'(x_0 + ph) = 0$. The usual methods for finding where the derivative of a function has a certain value are either (1) to calculate the derivatives at the tabular points and then perform inverse interpolation, (2) to estimate where the derivative has the desired value and then subtabulate the first derivative in which connection a table by SALZER[1] will be found helpful, or (3) to reverse the series in p, obtained by differentiating any one of the direct interpolation formulas for $f(x_0 + ph)$. The drawback in (1) is that it involves two separate steps, and the drawback in (2) is that it requires much more work than is necessary, together with a large amount of continual personal inspection. Thus method (3) which gives p in terms of the tabular entries $f(x_0 + mh)$ seems to be the most convenient, and it is recommended by GIBB,[2] in the form of reversion of a series involving differences. This present note gives the coefficients of the reversed series for p directly in terms of the tabular entries, so that no differences are necessary. However, if differences are already available, one might use them conveniently in the formulas given at the end which are obtained by reversing the series for the derivative of the Newton-Stirling formula using central differences δ_0^{2m} and mean-central differences $\mu\delta_0^{2m+1}$, viz.,

$$hf'(x_0 + ph) = (\mu\delta_0 - \tfrac{1}{6}\mu\delta_0^3 + \tfrac{1}{30}\mu\delta_0^5 - \cdots) + p(\delta_0^2 - \tfrac{1}{12}\delta_0^4 + \tfrac{1}{90}\delta_0^6 - \cdots)$$
$$+ p^2(\tfrac{1}{2}\mu\delta_0^3 - \tfrac{1}{8}\mu\delta_0^5 + \cdots) + p^3(\tfrac{1}{6}\delta_0^4 - \tfrac{1}{36}\delta_0^6 + \cdots)$$
$$+ p^4(\tfrac{1}{24}\mu\delta_0^5 - \cdots) + p^5(\tfrac{1}{120}\delta_0^6 - \cdots) + \cdots$$

which, after transposition of the constant term in the right member, and division by the coefficient of p, assumes the form $r = p + sp^2 + tp^3 + up^4 + vp^5 + \cdots$. The formulas are for all cases where 3 through 7 points are needed in direct interpolation. These formulas are similar to the formulas for inverse interpolation by the author;[3] but these combine the two operations of differentiation and inverse interpolation in a simple manner.

The function $f(x)$ requires n points for direct interpolation when the interpolating polynomial which equals $f(x_0 + mh) \equiv f(x_m) \equiv f_m$, $x_m \equiv x_0 + mh$, $m = -[(n-1)/2]$ to $[n/2]$, is of the $(n-1)^{\text{th}}$ degree. Corresponding to $n = 3(1)7$, quantities r, s, t, u and v are defined below. (The user must note carefully that in $f_x' \equiv f'(x)$, the derivative is with respect to x, and not p.)

3-Point

$r = (2hf_x' + f_{-1} - f_1)/D$, $s = t = u = v = 0$,

where $D = 2(f_{-1} - 2f_0 + f_1)$.

4-Point

$r = (6hf_x' + 2f_{-1} + 3f_0 - 6f_1 + f_2)/D$,

$s = 3(-f_{-1} + 3f_0 - 3f_1 + f_2)/D$, $t = u = v = 0$,

where $D = 6(f_{-1} - 2f_0 + f_1)$.

5-Point

$$r = (24hf_x' + 2(-f_{-2} + 8f_{-1} - 8f_1 + f_2))/D,$$
$$s = 6(-f_{-2} + 2f_{-1} - 2f_1 + f_2)/D,$$
$$t = 4(f_{-2} - 4f_{-1} + 6f_0 - 4f_1 + f_2)/D,$$
$$u = v = 0,$$

where $D = 2(-f_{-2} + 16f_{-1} - 30f_0 + 16f_1 - f_2)$.

6-Point

$$r = (120hf_x' + 2(-3f_{-2} + 30f_{-1} + 20f_0 - 60f_1 + 15f_2 - 2f_3))/D,$$
$$s = 15(-f_{-2} - f_{-1} + 10f_0 - 14f_1 + 7f_2 - f_3)/D,$$
$$t = 20(f_{-2} - 4f_{-1} + 6f_0 - 4f_1 + f_2)/D,$$
$$u = 5(-f_{-2} + 5f_{-1} - 10f_0 + 10f_1 - 5f_2 + f_3)/D,$$
$$v = 0,$$

where $D = 10(-f_{-2} + 16f_{-1} - 30f_0 + 16f_1 - f_2)$.

7-Point

$$r = (720hf_x' + 12(f_{-3} - 9f_{-2} + 45f_{-1} - 45f_1 + 9f_2 - f_3))/D,$$
$$s = 45(f_{-3} - 8f_{-2} + 13f_{-1} - 13f_1 + 8f_2 - f_3)/D,$$
$$t = 20(-f_{-3} + 12f_{-2} - 39f_{-1} + 56f_0 - 39f_1 + 12f_2 - f_3)/D,$$
$$u = 15(-f_{-3} + 4f_{-2} - 5f_{-1} + 5f_1 - 4f_2 + f_3)/D,$$
$$v = 6(f_{-3} - 6f_{-2} + 15f_{-1} - 20f_0 + 15f_1 - 6f_2 + f_3)/D,$$

where $D = 4(2f_{-3} - 27f_{-2} + 270f_{-1} - 490f_0 + 270f_1 - 27f_2 + 2f_3)$.

Then the quantity p is given by

$$p = r - r^2s + r^3(2s^2 - t) + r^4(-5s^3 + 5st - u) + r^5(14s^4 - 21s^2t + 3t^2 + 6su - v) + r^6(-42s^5 + 84s^3t - 28st^2 - 28s^2u + 7tu + 7sv) + \cdots.$$

When differences are available it is convenient to define the r, s, t, u and v according to the scheme below, the formula for p being the same as above. However, these formulas using differences, when expressed in terms of the tabular entries f_m, are not the same as those previously given directly in terms of f_m. Thus, when differences are available, these formulas using differences might serve as a partial check on those formulas without differences, or vice versa:

3-Point

$$r = (hf_x' - \mu\delta_0)/\delta_0^2,\ s = t = u = v = 0.$$

4-Point

$$r = (hf_x' - \mu\delta_0 + \tfrac{1}{6}\mu\delta_0^3)/\delta_0^2,$$
$$s = \tfrac{1}{2}\mu\delta_0^3/\delta_0^2,\ t = u = v = 0.$$

5-Point

$r = (hf_x' - \mu\delta_0 + \frac{1}{6}\mu\delta_0^3)/\Delta,$

$s = \frac{1}{2}\mu\delta_0^2/\Delta,$

$t = \frac{1}{6}\delta_0^4/\Delta,\ u = v = 0,$

where $\Delta = \delta_0^2 - \frac{1}{12}\delta_0^4.$

6-Point

$r = (hf_x' - \mu\delta_0 + \frac{1}{6}\mu\delta_0^3 - \frac{1}{30}\mu\delta_0^5)/\Delta,$

$s = (\frac{1}{2}\mu\delta_0^3 - \frac{1}{8}\mu\delta_0^5)/\Delta,$

$t = \frac{1}{6}\delta_0^4/\Delta,$

$u = \frac{1}{24}\mu\delta_0^5/\Delta,\ v = 0,$

where $\Delta = \delta_0^2 - \frac{1}{12}\delta_0^4.$

7-Point

$r = (hf_x' - \mu\delta_0 + \frac{1}{6}\mu\delta_0^3 - \frac{1}{30}\mu\delta_0^5)/\Delta,$

$s = (\frac{1}{2}\mu\delta_0^3 - \frac{1}{8}\mu\delta_0^5)/\Delta,$

$t = (\frac{1}{6}\delta_0^4 - \frac{1}{30}\delta_0^6)/\Delta,$

$u = \frac{1}{24}\mu\delta_0^5/\Delta,$

$v = \frac{1}{120}\delta_0^6/\Delta,$

where $\Delta = \delta_0^2 - \frac{1}{12}\delta_0^4 + \frac{1}{90}\delta_0^6.$

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[1] H. E. SALZER, "Table of coefficients for obtaining the first derivative without differences," NBS, *Applied Mathematics Series No. 2*, 1948.

[2] D. GIBB, *A Course in Interpolation and Numerical Integration for the Mathematical Laboratory*. London, Bell, 1915.

[3] H. E. SALZER, "A new formula for inverse interpolation," Amer. Math. Soc., *Bull.*, v. 50, 1944, p. 513–516

RECENT MATHEMATICAL TABLES

915[F].—K. FRÜCHTL, "Statistische Untersuchung über die Verteilung von Primzahl-Zwillingen," Öster. Akad. Wiss., *math.-nat. Kl., Anz.*, 1950, p. 226–232.

Information on the distribution of twin primes $(p, p + 2)$ and quadruplets $(p, p + 2, p + 6, p + 8)$ is given for the first 1020000 natural numbers. The information is based on the old table of CHERNAC.[1] The main table gives the number of prime pairs in each of the 1020 chileads $1000n < p < 1000(n + 1)$ for $n = 0(1)1019$. There is no chilead devoid of prime pairs and only three ($n = 688{,}851{,}927$) with but a single prime pair. The rows of this table are added to give the number of prime pairs in each of the 102 myriads $10000n < p < 10000(n + 1)$ for $n = 0(1)101$. These frequencies are, in turn, added by tens to give a 10 entry table for each interval of 100000. The grand total gives 8168 prime pairs $< 10^6$.

As for the quadruplets, there are enumerated in each of the 10 intervals of 100000; the total number is 166. On p. 232 the largest quadruplet, $p + 8$, is given for each of the 172 cases below 1020000.

Previous tables concerning twin primes are mentioned in *MTAC*, v. 4, p. 84. The reviewer has not as yet attempted to reconcile discrepancies between these and the tables under review. No doubt some of these are due to errata in Chernac.

D. H. L.

[1] L. CHERNAC, *Cribrum Arithmeticum* etc., Deventer, 1811.

916[I, O].—R. E. BEARD, "Some notes on approximate product integration," Inst. of Actuaries, *Jn.*, v. 73, part II, 1948, 356–403.

The author studies approximation formulas of the form:

$$\int_a^b f(x)\phi(x)dx = [\sum_{r=1}^{n} a_r f(x_r)] \cdot \int_a^b \phi(x)dx + R_n \int_a^b \phi(x)dx,$$

$a \leq x_1, x_2, \cdots, x_n \leq b$, where the a_r, x_r are independent of $f(x)$ and $\int_a^b \phi(x)dx \neq 0$. $\phi(x)$ is generally a known, tabulated function and $f(x)$ is a complicated or empirically given function. One of the more important applications of such formulas is the evaluation of continuous single premiums in the field of life and disability contingencies where $\phi(x) = v^x$ is the discount factor and $f(x)$ is derived from a mortality or disability table for one or more lives.

The paper is divided into two parts: a theoretical part in which formulas are developed for the determination of a_r, x_r and the remainder term R_n, and a part containing tables for the a_r and x_r based on the formulas of the first part for low orders (i.e. small n).

In the theoretical part the following cases are considered:

(1) Neither the a_r nor the x_r are given; R_n depends on $f^{(2n)}(x)$.

(2) $a_1 = a_2 = \cdots = a_n = \dfrac{1}{n}$; R_n depends on $f^{(n+1)}(x)$.

(3) $n = 3$, $a_1 = a_3$; R_3 depends on $f^{(3)}(x)$.

(4) The x_r are given, R_n depends on $f^{(n)}(x)$.

If $m_i = \int_a^b x^i\phi(x)dx / \int_a^b \phi(x)dx$ are the moments of $\phi(x)$ which are assumed to exist and to be known, the x_r in the cases (1) and (2) are found to be the roots of a polynomial of degree n whose coefficients are determinants the elements of which are multiples of the moments m_i. In the cases (1) and (4) the a_r are expressed as quotients of two determinants whose elements depend in a simple manner on the x_r and the m_i. Simpson's rule, Weddle's and Hardy's formulas are examples of formulas belonging to case (4).

In deriving his formulas for the remainder term R_n the author makes an application of the mean value theorem which is correct only if $\phi(x)$ is non-negative and $0 \leq a < b$ and arrives thereby at an expression for R_n of the form $f^{(k)}(\xi_1)\dfrac{m_r}{k!} - f^{(k)}(\xi_2)h(x_1, \cdots, x_r)$ where k has the values indicated above, $h(x_1, \cdots, x_r)$ is a function of the x_r only which in many cases can

be expressed as a simple function of the m_i, and $0 < \xi_1, \xi_2 < b$. He then simplifies this expression by stating that, for practical purposes, ξ_1 and ξ_2 may be replaced by a common value with $0 < \xi < b$ which is evidently correct only in particular cases. The remainder terms given by the author are therefore useless in many cases for the purpose of measuring the accuracy of the approximation formulas. It is, in any event, doubtful whether remainder terms expressed by means of derivatives of high order are useful for empirically given functions $f(x)$.

The author proceeds to develop from the general formulas the formulas corresponding to low values of n. In many cases he first makes a linear transformation of the variable x to a variable X so as to make $m_1 = 0$ and $m_2 = 1$ and then expresses the remaining moments by means of Pearson's β_i. In those formulas where β_i for $i \geq 3$ appear he makes the further assumption that the β_i for $i \geq 3$ can be expressed as functions of β_1 and β_2 in the same manner as for Pearsonian distribution functions.

In another special application the author considers special functions $\phi(x)$ such as $\phi(x) = 1$, $e^{-\lambda x}$, $\exp(-\{(x-m)/c\}^2)$, $(1-x)^{m_1}(1+x)^{m_2}$.

No attempt has been made to classify the various formulas by some optimum properties or by the magnitude of the remainder terms, the only test being a comparison of the true values of some actuarial functions with the results of various approximation formulas.

In the numerical part of the paper the following tables are given:

Table 1: Solutions to 6D for X_1, X_2, a_1, a_2 for the two point formulas of case (1) corresponding to $\beta_1 = 0(.1)3.0(.1)3.0$.

Table 2: Solutions to 6D for X_1, X_2, X_3, a_1, a_2, a_3 for the three point formula of case (1) corresponding to $\beta_1 = 0(.2)2$ and $\beta_2 = 2(.5)6$.

Table 3: Solutions to 6D for X_1, X_2, X_3 for the three point formula of case (2) corresponding to $\beta_1 = 0(.01).50$.

Table 4: Solutions to 6D for X_1, X_2, X_3, $a_1 = a_3 = \dfrac{1}{2+A}$, $a_2 = \dfrac{A}{2+A}$ for case (3) corresponding to $\beta_1 = 0(.2)2$ and $\beta_2 = 2.0(.5)6.0$.

Table 5: Moments m_1, m_2, m_3, σ to 3D and $\sqrt{\beta_1}$ to 5D for the continuous function $(1+i)^{-n}$ for $i = .02(.01).06$ and $n = 5(5)60$.

Table 6: A similar table for discrete moments.

Table 7: Solutions to 5D for a_1, a_2, a_3 for the three point formula of case (4), where $x_1 = 0$, $x_2 = \dfrac{m}{2}$, $x_m = m$ and $\phi(x) = (1+i)^{-x}$ for $m = 5(5)50$ and $i = .02(.01).06$.

Table 8: The values to 3D of $\bar{e}_x$, $\bar{e}_{xx}$, $\bar{e}_{xxx}$, $\bar{e}_{xxxx}$, $\bar{e}_{xxxxx}$ for the A 1924–29 ultimate mortality tables for $x = 15(1)80$.

Table 9: σ to 3D, $(\beta_1)^{\frac{1}{2}}$ to 5D, β_2 to 4D for the function $(\mu\bar{l})_{xx\ldots}$ for 1 to 5 lives of the A 1924–29 ultimate mortality table.

There are also a few auxiliary tables for the purpose of evaluating remainder terms.

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917[I].—LEROY F. MEYERS & ARTHUR SARD, "Best approximate integration formulas," *Jn. Math. Phys.*, v. 29, 1950, p. 118–123.

For each $A = c_0x(0) + c_1x(1) + \cdots + c_mx(m)$ that is an approximation to $\int_0^m x(t)\,dt$ which is exact whenever $x(t)$ is a polynomial in t of degree n, $m \geq 1$, $n \geq 0$, there is a kernel function $k(t)$ such that, when $x(t)$ is of class C^{n+1},

$$R[x] \equiv \int_0^m x(t)\,dt - A = \int_0^m x^{(n+1)}(t)k(t)\,dt.$$

The kernel function is defined explicitly by

$$k(t') = R[\psi_{t'}] = -R[\phi_{t'}]$$

where

$$\psi_{t'} = \psi_{t'}(t) = \begin{cases} 0 \\ (t-t')^n/n!, \end{cases} \qquad \phi_{t'} = \phi_{t'}(t) = \begin{cases} (t-t')^n/n! & \text{if } t \leq t' \\ 0 & \text{if } t > t'. \end{cases}$$

By Schwarz's inequality

$$|R[x]| \leq \left(\int_0^m k^2(t)\,dt\right)^{\frac{1}{2}} \left(\int_0^m x^{(n+1)}(t)^2 dt\right)^{\frac{1}{2}}.$$

The *best* approximation A, for given m and n, is defined as that which minimizes $J \equiv \int_0^m k^2(t)\,dt$.

In the present paper, the authors give the best integration formulas for $n = 1$, $m = 1(1)20$; $n = 2$, $m = 2(1)12$; $n = 3$, $m = 2(1)9$. For these values of n and m, they tabulate the exact values of $c_0, c_1, \cdots, c_m$ and J. In an earlier paper by SARD[1] the best integration formulas are given for $n = 0$, all m, and $n = 1, 2, 3$, $m \leq 6$.

The authors derive some fundamental algebraic relationships between the c_i's and J. Then for the case $n = 1$, recursive relations are derived which afford a complete characterization of the best integration formulas for any m. Finally, the authors give some conjectures about the convergence of the coefficients, some of which are true for $n = 0$ and 1, but which are open questions for $n \geq 2$.

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[1] A. SARD, "Best approximate integral formulas; best approximation formulas," *Amer. Jn. Math.*, v. 71, 1949, p. 80–91.

918[I].—LEROY F. MEYERS & ARTHUR SARD, "Best interpolation formulas," *Jn. Math. Phys.*, v. 29, 1950, p. 198–206.

The tabular values $x(0)$, $x(1)$, $x(2)$ of a function $x(t)$ being given, the problem discussed here is the approximation of $x(u)$ by an expression of the form $A = a_0x(0) + a_1x(1) + \cdots + a_mx(m)$, where the value of m and the coefficients $a_0 = a_0(u)$, $a_1 = a_1(u)$, $\cdots$, $a_m = a_m(u)$ are to be determined. If A is an exact approximation whenever $x(t)$ is a polynomial of degree n, there is a kernel function $k(t, u)$, such that when $x(t)$ is of class C^{n+1},

$$R[x] = x(u) - A = \int_K x^{(n+1)}(t)k(t, u)\,dt,$$

where K is the smallest interval containing u and those values of $0, 1, \cdots, m$ for which the corresponding $a_0, a_1, \cdots, a_m$ are not zero. For each u, $k(t, u)$ is a broken polynomial in t consisting of at most $m + 3$ arcs, which is defined explicitly by

$$k(t', u) = R[\psi_{t'}] = -R[\phi_{t'}],$$

where

$$\psi_{t'} = \psi_{t'}(t) = \begin{cases} 0 \\ (t - t')^n/n!, \end{cases} \qquad \phi_{t'} = \phi_{t'}(t) = \begin{cases} (t - t')^n/n! & \text{if } t \leq t', \\ 0 & \text{if } t > t'. \end{cases}$$

By Schwarz's inequality

$$|R[x]| \leq M\left[\int_K x^{(n+1)}(t)^2 dt/|K|\right]^{\frac{1}{2}}$$

where the modulus M is defined by $M = M(u) = [|K| \int_K k(t, u)^2 dt]^{\frac{1}{2}}$. For given m, n, u, that A is called *best* which minimizes the modulus M, and it is denoted by $A_{m,n,u}$. The authors report that for $n = 0$, all m, and $n = 1$, $m = 1(1)4$, $n = 2$, $m = 2$, the conventional polynomial interpolation is best, but not for $n = 2$, $m = 3, 4, 5$.

The determination of $A_{m,n,u}$ involves the prior determination of a number of approximations $B_{m,n,u}$ (for different values of m and u) where $B_{m,n,u}$ is that A which minimizes $J \equiv \int_{\min(u,0)}^{\max(u,m)} k(t, u)^2 dt$. The modulus of $B_{m,n,u}$ is denoted by $M_{m,n,u}$.

The authors tabulate the auxiliary function $\mu = M^2_{m,2,u}/\theta(u)$ where $\theta(u) = (u - [u])^2(1 - u + [u])^2/120$ for $m = 2(1)5$, $u = 0(.1)2$; $m = 3(1)5$, $u = 2.1(.1)2.5$; 2D. The table of μ enables one (a) to compare moduli, (b) to identify $A_{m+j,2,u}$ with $B_{m,2,u}$, $0 \leq 5 - m$, where each different value of m corresponds to a different range of u, and (c) to find the proper argument by translation in using the main table.

The principal table in the article is the collection of formulas for $B_{m,2,u}$, $0 \leq u \leq [(m + 1)/2]$, $m = 2, 3, 4$, and $M^2_{m,2,u}$, $u \leq [(m + 1)/2]$, $m = 2, 3, 4, 5$. In the $B_{m,2,u}$, the coefficients of $x_0, x_1, \cdots, x_m$ are given as exact polynomials in u. The $M^2_{m,2,u}$ are expressed as either exact polynomials in u, or as exact polynomials in u multiplied by $\theta(u)$. The expressions for $B_{m,2,u}$ and $M^2_{m,2,u}$ are different for u lying within different ranges.

The rest of the paper is concerned with the derivation of those formulas and their transformation under a linear transformation of the t-axis, $t^* = bt + c$, $b \neq 0$.

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NBSCL

919[J].—T. M. CHERRY. "Summation of slowly convergent series," Camb. Phil. Soc., *Proc.*, v. 46, 1950, p. 436–449.

This paper studies two transformations of the remainder that are helpful in numerical summations.

The power series considered is $\sum_0^\infty C_r t^r$, and, in the remainder $\sum_n^\infty C_r t^r$, C_r is written as a product $c_r f(r)$, where it is supposed that $f(z)$ has an asymptotic expansion

$$f(z) \sim Az^\alpha + A_1 z^{\alpha_1} + A_2 z^{\alpha_2} + \cdots$$

with appropriate conditions. Then, if

$$c_n + c_{n+1}t + c_{n+2}t^2 + \cdots = \phi_n(t)$$

and

$$\vartheta = t\frac{d}{dt}, \quad D_t = \frac{d}{dt}, \quad D_n = \frac{d}{dn}, \quad \Delta f(n) = f(n+1) - f(n),$$

the transformations are

$$\sum_{r=0}^{\infty} c_{n+r}t^{n+r}f(n+r) = t^n \sum_{r=0}^{p-1} \vartheta^r\phi_n(t)\cdot D_n{}^r f(n)/r! + R_{n,p}$$

$$\sum_{r=0}^{\infty} c_{n+r}t^{n+r}f(n+r) = t^n \sum_{r=0}^{p-1} t^r D_t{}^r\phi_n(t)\cdot \Delta^r f(n)/r! + S_{n,p}$$

where $R_{n,p}$, $S_{n,p}$ are "error-terms."

The assumption of an asymptotic expansion for $f(z)$ is needed for the theoretical development, although its coefficients do not occur in the numerical applications. On the other hand, although $\phi_n(t)$ is not, in theory, very restricted, it must be easily accessible numerically which means, virtually, that it must be chosen in such a way that a closed expression for it is known.

A thorough discussion of the error terms $R_{n,p}$, $S_{n,p}$ is given, and upper bounds for them are obtained. In practice these upper bounds exceed the true error considerably, so that a closer rough estimate has been sought which is more useful practically, although it gives the order of the error only; for the numerical examples tested the true error rarely exceeds the estimate, and never by a factor exceeding about 1.2.

Two particular families of functions $\phi_n(t)$ are studied in considerable detail, and are used in the numerical examples. These are binomial remainder functions.

The first family is defined by

$$B_{0,n}(t) = B_0(t) = 1 + t + t^2 + \cdots = (1-t)^{-1}$$

$$B_{1,n}(t) = \frac{1}{n} + \frac{t}{n+1} + \frac{t^2}{n+2} + \cdots = t^{-n}\left\{-\ln(1-t) - t - \tfrac{1}{2}t^2 - \cdots - \frac{t^{n-1}}{n-1}\right\}$$

$$B_{2,n}(t) = \frac{1}{n(n+1)} + \frac{t}{(n+1)(n+2)} + \cdots = \frac{1}{t}\left\{\frac{1}{n} - (1-t)B_{1,n}(t)\right\}$$

$$B_{3,n}(t) = \frac{1}{n(n+1)(n+2)} + \frac{t}{(n+1)(n+2)(n+3)} + \cdots = \frac{1}{2t}\left\{\frac{1}{n(n+1)} - (1-t)B_{2,n}(t)\right\}$$

and so on.

The second family, with

$$b_n = \frac{1}{2}\cdot\frac{3}{4}\cdots\frac{2n-1}{2n} = \frac{2n-1}{2n}\,b_{n-1}$$

is given by

$$\mathbf{B}_{k,n}(t) = b_n + b_{n+1}t + b_{n+2}t^2 + \cdots = t^{-n}\{(1-t)^{-k} - 1 - kt - \cdots - b_{n-1}t^{n-1}\}$$

$$\mathbf{B}_{k,n}(t) = \frac{b_n}{n+1} + \frac{b_{n+1}t}{n+2} + \cdots = \frac{2}{t}\{b_n - (1-t)\mathbf{B}_{k,n}(t)\}$$

$$\mathbf{B}_{k,n}(t) = \frac{b_n}{(n+1)(n+2)} + \frac{b_{n+1}t}{(n+2)(n+3)} + \cdots = \frac{2}{3t}\left\{\frac{b_n}{n+1} - (1-t)\mathbf{B}_{k,n}(t)\right\}$$

and so on.

Tables are given of $\mathbf{B}_{k,10}(t) = \sum_{s=0}^{\infty} b_{10+s}t^s = R(r, \theta) + iI(r, \theta)$ to 4 decimals for $r = 0.7(0.05)1$, $\theta = 0°(5°)90°$, and have been used for the numerical examples, which concern the Kapteyn series

$$\sum_{r=1}^{\infty} x^r J_r(ry)$$

and are very fully considered.

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920[K, L].—B. V. GNEDENKO, *Kurs Teorii Veroiâtnostei* [A Course in the Theory of Probabilities]. Moscow and Leningrad, 1950, 388 p.

On p. 372–385 there are tables of

$$\phi(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2}, \quad x = [0(.01)3.99; 4D]$$

$$\Phi(x) = \frac{1}{\sqrt{2\pi}}\int^x e^{-x^2/2}dx, \quad x = 0(.01)2(.02)3(.2)4(.5)5;\ 4D \text{ up to } 2.98,$$

5D to 8D thereafter

$$P_k(a) = \frac{a^k e^{-a}}{k!}, \quad a = .1(.1)1(1)9, \quad k = [0(1)27; 6D]$$

$$\sum_{m=0}^{k}\frac{a^m(e^{-a})}{m!}, \quad a = .1(.1)1(1)3, \quad k = [0(1)15; 6D]$$

$$P(x) = \frac{1}{2^{(k-2)/2}\Gamma\left(\frac{k}{2}\right)}\int_x^\infty z^{k-1}e^{-z^2/2}dz, \quad x = 1(1)30, \quad k = [1(1)29; 4D]$$

$$S(x) = \frac{\Gamma\left(\frac{n}{2}\right)}{((n-1)\pi)^{\frac{1}{2}}\Gamma\left(\frac{n-1}{2}\right)}\int_{-\infty}^{x}\left(1 + \frac{z^2}{n-1}\right)^{-n/2}dz$$

$$n = 2(1)20, \infty, \quad x = [0(.1)6; 3D], \quad x = \infty; \ 5D$$

$$K(x) = \sum_{k=-\infty}^{\infty} (-1)^k e^{-2k^2x^2}, \quad x = .28(.01)2.50(.05)3; \text{ mostly 6D}$$

$$\ln \frac{1-\beta}{\alpha} \qquad \alpha = .001, .01(.01).05, .1, .15$$
$$\beta = [.001, .01(.01).05, .1, .15; 3D]$$

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921[L].—AIRCRAFT RADIATION SYSTEMS LABORATORY, "Tables of modified cosine-integral," Stanford Research Institute, 1951, viii + 56 p.

These tables contain 6D values of

$$\overline{\text{Ci}}\,(x) = \int_0^x \frac{1-\cos t}{t}\,dt$$

for $x = 0(.001)10(.01)50$. The computation was carried out, largely on IBM machines, by the Telecomputing Corporation of Burbank, California.

The introduction, by C. T. TAI, discusses the connection of $\overline{\text{Ci}}$ with the cosine-integral function and the application of the tables, and describes the computation and preparation of the tables. A bibliography is appended.

The work was sponsored by the U. S. Air Force.

A. E.

922[L, M, Q, S].—M. P. BARNETT & C. A. COULSON, "The evaluation of integrals occurring in the theory of molecular structure. Parts I and II," Roy. Soc. London, *Philos. Trans.*, v. 243A, 1951, p. 221–249.

Table 1, p. 233. Modified Bessel functions of the first kind, $I_{n+\frac{1}{2}}(x)$, to 7S for $n = -1(1)4$ and $x = .5(.5)10$.

Table 2, p. 233. Modified Bessel functions of the third kind, $K_{n+\frac{1}{2}}(x)$, to 7S for $n = -1(1)4$ and $x = .5(.5)10(1)25$.

These tables are used for the numerical computation of the functions ζ defined by the expansion

$$r^{m-1}e^{-r} = (t\tau)^{-\frac{1}{2}} \sum_{n=0}^{\infty} (2n+1)P_n(\cos\theta)\zeta_{m,n}(1, t; \tau)$$

where $r^2 = t^2 + \tau^2 - 2t\tau\cos\theta$. With these functions the authors form

$$\mathbf{Z}_{m,n,l+\frac{1}{2}}(\kappa, \tau) = \int_0^{\infty} e^{-\kappa t}\zeta_{m,n}(1, t; \tau)t^{l+\frac{1}{2}}dt$$

and discuss the computation of **Z** by both numerical integration and analytical methods.

In the memoir it is shown that a large number of integrals occurring both in nuclear physics and astrophysics can be reduced to known integrals and to **Z** integrals. Formulas are listed for more than 180 integrals.

A. E.

923[L].—C. L. BARTBERGER, "The magnetic field of a plane circular loop," *Jn. Appl. Phys.*, v. 21, 1950, p. 1108–1114.

The integrals

$$I_1 = \pi^{-1} \int_0^\pi (1 - b \cos \theta)^{-1} d\theta$$

$$I_2 = \pi^{-1} \int_0^\pi (1 - b \cos \theta)^{-1} \cos \theta d\theta$$

are expressed in terms of complete elliptic integrals. Series expansions are also given in ascending powers of b and $1 - b$.

Table I (p. 1110–1111) gives 6D values of I_1, and table II (p. 1111–1112) 6D values of I_2, for $b = 0(.001).809$.

Table III (p. 1113–1114) gives 6D values of I_1, I_2, $I_1 - I_2$, $(1 - b)I_1$, $(1 - b)I_2$ for $b = .8(.001)1$, and table IV (p. 1114) 6D values of the same functions as table III for $b = .995(.0001)1$.

A. E.

924[L].—A. FLETCHER, "Tables of two integrals and of Spielrein's inductance function," *Quart. Jn. Mech. Appl. Math.*, v. 4, 1951, p. 223–235.

The tables (p. 229–232) are of

$$I = \int_a^1 (K - E) dk, \qquad J = \int_a^1 (K - E) k^{-3} dk$$

and $-16\pi(I - \alpha^3 J)/[3(1 - \alpha)^2]$.

Values are given to 10D, 10D and 6D respectively. The range of α is $0(.01)1$. Half a dozen small auxiliary tables are also given.

925[L].—Harvard University, COMPUTATION LABORATORY, *Annals*, v. 14: *Tables of the Bessel Functions of the First Kind of Orders Seventy-Nine through One Hundred Thirty-Five.* Cambridge, Mass., Harvard University Press, 1951, viii, 614 p. 19.5 × 26.7 cm. $8.00.

This is the twelfth and final volume of the monumental set of Tables of Bessel Functions of the First Order, published by Harvard during the past five years—six volumes in 1947, three in 1948, two in 1949 and one in 1951. The tabular parts of the volumes fill 7652 pages. The previous 11 volumes have been reviewed in *MTAC:* v. 2, p. 261–262, 344; v. 3, p. 102, 185–186, 367, 474–475; v. 4, p. 22, 92. Roughly speaking we now have here 10D tables of all $J_n(x)$, for $x = 0(.01)100$, when $n = 0(1)111$; and for $x = 0(.1)$-100, when n has any positive integral value > 111; for $n > 135$ the values of $J_n(x)$ are always less than 10^{-10}. In addition to what is thus stated, for $n = 0(1)3$, $x = [0(.001)25(.01)100; 18D]$; for $n = 4(1)15$, $x = [0(.001)$-$25(.01)100; 10D]$. Detailed information concerning interpolation in the whole range is given in *Annals*, v. 3 and 5; for 10D interpolation the work is not excessive.

Further, in the present volume we have 10D tables of $J_n(n)$ for $n = 0(1)$-100. Zero values are not given, since the values found by the Computation Laboratory were presented to the Royal Society Committee for use in connection with their second volume of Bessel function tables. For $n \geq 92$, $x \leq 100$, $J_n(x) \neq 0$. For $J_n(n)$, in the Harvard range, we had earlier: MEISSEL (1891), $n = 20$ to 20D; MEISSEL (1895), $n = 1(1)24$ to 18D; AIREY (1916),

$n = 1(1)50(5)100$ to 6D; WATSON (1922), $n = 1(1)50$ to 7D; and HAYASHI (1930), $n = 2$ to 101D, $n = 10$ to 61D, $n = 20$ to 41D, $n = 30$ to 35D, $n = 40$ to 35D, $n = 50$ to 30D, $n = 100$ to 18D. Hence most of the values in this special Harvard table are new.

In the recent Russian table of FADDEEVA and GAVURIN, RMT **852**, the argument extends to 124.9, at interval .1, so that some 6D of $J_n(x)$ for all orders, $n = 0(1)120$ supplement values given in the Harvard tables. So also for 5D zeros < 125, of $J_n(x)$; the last zero is for $J_{115}(x)$.

The remarkable Automatic Sequence-Controlled Calculator on which these tables were computed carried the values of $J_n(x)$, for $n = 0(1)3$ to 23D, and for $n > 3$ to not less than 13D and most of the time much more than this; its electromagnetic typewriters also wrote out, for checking purposes, 10 differences in every case, and also finally produced the 18D or 10D copy which could be sent directly to the printer for offset reproduction. The computation of these tables was only a tiny fraction of the work achieved in the ASCC since its activities began in 1945, and have continued to the present, 24 hours a day, 7 days a week.

Only one tabular slip has ever been found in the published volumes, $J_3(72.10)$ [*MTAC*, v. 3, p. 41], but this slip was due to some reproduction difficulty, and not to any error in computation or in automatic mechanical checking. In the last line of the second page of the "Preface," of the volume under review, *for* $J(x)$, $n = 0(1)120$, $x = [0(.01)14.99; 8D]$, *read* $J_n(x)$, $n = 0(1)120$, $x = [0(.1)124.9; 6D]$; $n = 0(1)13$, $x = [0(.01)14.99; 8D]$.

In the first page of the Preface, line -2, *for* Claire, *read* Clare.

These Harvard Bessel Function tables, with most of the values new, constitute an outstanding contribution to scientific research.

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926[L, S].—HELMAR KRUPP, "Bestimmung der allgemeinen Lösung der Schrödinger-Gleichung für Coulomb-Potential," Akad. Wiss. Leipzig, *mat.-phys. Kl., Berichte*, v. 97, 1950, no. 8, p. 1–28.

The Schrödinger equation for the Coulomb potential is

$$\frac{d^2R}{dr^2} + \frac{2}{r}\frac{dR}{dr} + 2\left[-\frac{1}{2n^2} + \frac{1}{r} - \frac{l(l+1)}{2r^2}\right]R = 0, \qquad l = 0, 1, 2, \cdots.$$

Two solutions are written in the form

$$_jR(n, l, r) = \frac{(2r)^l}{(2l+1)!}\, e^{-r/n}\, {}_jM(a, b, x), \qquad j = 1, 2,$$

where $a = l + 1 - n$, $b = 2l + 2$, $x = 2r/n$.

With the abbreviations

$$(a)_0 = 1, \ (a)_m = a(a+1)\cdots(a+m-1) \text{ for } m = 1, 2, \cdots$$
$$A = \Psi(-a) \text{ if } a = 0, -1, -2, \cdots$$
$$A = \Psi(a-1) \text{ if } a \neq b-1, b-2, \cdots$$

the definitions of the M are

$$ {}_1M(a, b, x) = \sum_{m=0}^{\infty} \frac{(a)_m x^m}{(b)_m m!} $$

$$ \pi\, {}_2M(a, b, x) = [\ln x + C - \Psi(b-1) + A]\, {}_1M(a, b, x) + \sum_{m=1}^{\infty} \frac{(a)_m x^m}{(b)_m m!} \sum_{k=0}^{m-1} \left(\frac{1}{a+k} - \frac{1}{b+k} - \frac{1}{1+k} \right) + \sum_{m=0}^{b-2} \frac{(-)^{m-b}}{m!\Gamma(a)} \Gamma(a-b+m+1) b!(b-m-2)! x^{m+1-b}. $$

Tables are given for ${}_1R$, d_1R/dr, ${}_2R$, d_2R/dr for $0 \leq x \leq 15$ (the interval in most cases is .5), for $l = 0$ and $n = \frac{1}{2}\left(\frac{1}{2}\right)\frac{7}{2}$, $l = 1$ and $n = \frac{3}{2}\left(\frac{1}{2}\right)\frac{7}{2}$, $l = 2$ and $n = \frac{5}{2}\left(\frac{1}{2}\right)\frac{7}{2}$. Graphs of these functions are added. A misprint occurs on page 14, line 3 from the bottom and on page 15, line 6; in both places $\binom{-\kappa+n+l+1}{\kappa}$ should be replaced by $\binom{n+l}{\kappa}$. However the formulas to which the author refers as the source of computation do not contain this misprint.

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927[L].—U. E. KRUSE & N. F. RAMSEY, "The integral $\int_0^\infty y^3 \exp(-y^3 + ix/y)\,dy$," *Jn. Math. Phys.*, v. 30, 1951, p. 40–43.

In several problems of theoretical physics there arise integrals which can be reduced to the real part, $I(x)$, or the imaginary part, $K(x)$, of the integral mentioned in the title. Table 2 of this paper gives 5D values of both these functions for $x = 0(.1).4(.2)8(.5)20$.

Convergent expansions, useful for $x < 3$, have been given by ZAHN,[1] and Table 1 of the present paper gives numerical values, to 7–9S, of the coefficients up to that of x^{14}. For larger x it is more convenient to use asymptotic series developed by LAPORTE,[2] and 6D values of the first six coefficients in these series are also given in the present paper. [The author remarks that in TORREY's paper[3] the recurrence formula for the coefficients contains a misprint, but Torrey's numerical values are in agreement with the author's.]

Table 2 was computed from these expansions, and V. E. CULLER assisted in the computation.

A. E.

[1] C. T. ZAHN, "Absorption coefficients for thermal neutrons," *Phys. Rev.*, v. 52, 1937, p. 67–71.
[2] O. LAPORTE, "Absorption coefficients for thermal neutrons," *Phys. Rev.*, v. 52, 1937, p. 72–74.
[3] H. C. TORREY, "Notes on intensities of radio frequency spectra," *Phys. Rev.*, v. 59, 1941, p. 293–299.

928[L, S].—N. METROPOLIS & J. R. REITZ, "Solutions of the Fermi-Thomas-Dirac equation," *Jn. Chem. Phys.*, v. 19, 1951, p. 555–573.

Solutions ψ are given of the equation

$$\frac{d^2\psi}{dx^2} = x(\epsilon + \psi^{\frac{1}{2}}x^{-\frac{1}{2}})^3 \qquad (\epsilon^3 = 3 \cdot 2^{-5} \cdot \pi^{-2}Z^{-2})$$

in terms of the variable $w = (2x)^{\frac{1}{2}}$. The tables are at intervals of .08 and extend until ψ becomes negative. The parameter Z takes on the 24 values

$$Z = 6\ (4)\ 14,\ 16,\ 18\ (4)\ 26,\ 29\ (4)\ 81,\ 84\ (4)\ 92$$

and there are 8 different initial slopes ψ'. Actually 2ψ is tabulated to 5D. The calculations were done on the ENIAC.

D. H. L.

929[L].—L. PRANDTL, with the assistance of F. VANDREY, "Flieszgesetze normalzäher Stoffe im Rohr," *Zeit. angew. Math. Mech.*, v. 30, 1950, p. 169–174.

Table 1. 4S table of

$$\varphi(a) = \tfrac{1}{2}\cosh a - \frac{1}{2a}\sinh a + \frac{1}{a^2}(\cosh a - 1)$$

for $a = 0(.1)5(.2)10$.

Table 2. 3D table of $\varphi(a\xi)/\varphi(a)$ for $\xi = 0(.2).6(.1)1$ and $a = 1(1)10(2)14$. Some of the values were obtained by interpolation: these are put in parentheses.

A. E.

930[L].—S. SILVER & W. K. SAUNDERS, "The radiation from a transverse rectangular slot in a circular cylinder," *Jn. Appl. Phys.*, v. 21, 1950, 745–749.

Table I gives values of ka, n, and ϕ_0 for which $H_0^{(2)\prime}(ka)/H_n^{(2)\prime}(ka) \leq .0001$ and $(n\phi_0)^{-1}\sin n\phi_0 \geq .9$.

Table II gives 4D values of

$$\left| \sum_{n=0}^{\infty} \frac{\epsilon_n i^n \cos n\phi}{\sin\theta H_n^{(2)\prime}(ka\sin\theta)} \Big/ \sum_{n=0}^{\infty} \frac{\epsilon_n i^n}{H_n^{(2)\prime}(ka)} \right|$$

for $ka = .8$, $\theta = 10°(10°)90°$, $\phi = 0°(10°)180°$. Here $\epsilon_0 = 1$, $\epsilon_n = 2$ if $n > 0$.

Table III is similar to table II except that $ka = 2.5$.

A. E.

931[L].—E. WOLF, "Light distribution near focus in an error-free diffraction image," Roy. Soc. London, *Proc.*, v. 204A, 1951, 533–548.

In the course of the work the function

$$Q_{2m}(v) = \sum_{s=0}^{2m} (-1)^s [J_s(v)J_{2m-s}(v) + J_{s+1}(v)J_{2m+1-s}(v)]$$

is introduced, where the J are Bessel functions of the first kind.

Table 1 (p. 541) gives 5D values of $Q_{2m}(v)$ for $v = 0(1)15$ and $m = 0(1)M(v)$ where

$$M(v) = v + 1 \text{ for } v \leq 5,\ M(6) = 6,\ M(7) = M(8) = 7,\ M(9) = 8,$$
$$M(10) = M(11) = 9,\ M(12) = 10,\ M(13) = M(14) = 11,$$
$$\text{and } M(15) = 12.$$

The results of some numerical computations involving these functions are given in form of diagrams.

A. E.

932[V].—E. GRUSCHWITZ, *Calcul approché de la couche limite laminaire en écoulement compressible sur une paroi conductrice de la chaleur.* Office National d'Études et de Recherches Aeronautiques (O.N.E.R.A.). Publication No. 47, Chatillon-sous-Bagneux, Seine. 1950, 39 p.

As a generalization of the KÁRMÁN-POHLHAUSEN[1] method to steady compressible flow, the velocity component u parallel to a wall $y = 0$ is approximated within the boundary layer by a quartic in $\eta = \text{const} \int_0^y \rho dy$ for a fixed x; moreover, the density ρ is expressed as a rational function of η. These assumptions lead to the system:

$$(\theta u_e/\nu_e)\, d\theta/dx = F_1(K) - (K/b_0)[2 - M_e^2 F_2(k)], \tag{1}$$

$$b_0 = (1 + .2025\, M_e^2)[1 + M_e^2 F_3(K)]/[1 + M_e^2 F_4(k)], \tag{2}$$

$$K = (\theta^2 b_0/\nu_e)\, du_e/dx, \tag{3}$$

with known initial conditions at $u_e = 0$ for the determination of K (which is a quintic in Pohlhausen's parameter λ), $b_0 = [\rho_e/\rho]_{y=0}$, and the momentum loss $\theta = \int_0^{y_e} (\rho u/\rho_e u_e)(1 - u/u_e)\, dy$. The velocity u_e, kinematic viscosity ν_e, Mach number M_e and density ρ_e are supposed known in the main stream as well as $y_e = [y]_{u=.99u_e}$, while the functions $F_i(K)$, which depend on the Prandtl number Pr, are tabulated to 3D ($i = 2, 3, 4$) or 4D ($i = 1$) for $K = .094(-.001) - .156$ [Pr $= .725$ ($i = 1, 2, 3, 4$) and Pr $= 1$ ($i = 2$)]. In addition the displacement $\delta^* = \int_0^{y_e} (1 - \rho u/\rho_e u_e)\, dy$ of the main stream from the wall is given by $\delta^* = \theta[b_0 f_1(K) + M_e^2 f_2(K)]$, the $f_i(K)$ being tabulated to 3D for the above range in K [Pr $= .725$].

In the appendix E. A. EICHELBRENNER describes an exact method and presents graphical results indicating that certain aerodynamical quantities (excluding the temperature) are equally well approximated by using Pr $= .725$ or the simpler value Pr $= 1$ [$F_3(K) = F_4(K)$] in Gruschwitz's method.

The tables on pages 13–16 have columns of values of K (as above), λ (generally to 4S), F_3, F_4, f_1, f_2, F_1, F_2, F_2 [Pr $= 1$]. There are also rows of values for $\lambda = \pm 12$, 7.0523, $K = -.157$ (λ, F_1, F_2, F_2 [Pr $= 1$]).

The identities $F_2 = .595 + F_3$ and $f_2 = .405 - F_3$ should hold throughout the table.

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[1] K. POHLHAUSEN, "Zur näherungsweisen Integration der Differentialgleichungen der laminaren Grenzschicht." *Zeit. angew. Math. Mech.*, v. 1, 1921, p. 252–268.

MATHEMATICAL TABLES—ERRATA

193.—Crédit Communal de Belgique, *Tables d'Intérêts et d'Annuités.* Brussels, 1950.

Table V, p. 110, $1/a_{\overline{8}|}$ .08 *for* 0.08328680 *read* 0.08328684.

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194.—J. W. Glover, *Tables of Applied Mathematics in Finance, Insurance and Statistics.* Ann Arbor, 1930.

Part III, p. 394–413, Table of nth derivative of $(2\pi)^{-\frac{1}{2}} \exp(-\frac{1}{2}t^2)$.

p.	t	n	*for*	*read*
399	1.14	8	−30.15397	−30.154014
403	2.16	8	13.42550	13.425483
403	2.44	8	8.04225	8.042232
407	3.32	7	0.85568	0.855664
409	3.70	8	−1.27557	−1.275600
413	4.60	8	0.20725	0.207281
413	4.80	8	0.16334	0.163369

W. L. Semon

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195.—N. R. Jørgensen, *Undersøgelser over Frekvensflader og Korrelation.* Copenhagen, 1916.

A recomputation of a part of Jørgensen's tables of the nth derivative of $(2\pi)^{-\frac{1}{2}} \exp(-\frac{1}{2}x^2)$ by the Automatic Sequence Controlled Calculator reveals the following errata.

x	n	*for*	*read*
.04	6	−5.9506639	−5.95066324
.10	6	−5.7762539	−5.77625459
.14	6	−5.5796134	−5.57961394
.16	6	−5.4581549	−5.45815434
.32	6	−3.9980943	−3.99809459
.34	6	−3.7642068	−3.76420646
.40	6	−3.0122141	−3.01241439
.80	6	2.2938198	2.29381943
.82	5	−2.0400226	−2.04002227
.88	6	3.0653608	3.06536044
1.00	6	3.8715312	3.87153159
1.02	5	−1.3734687	−1.37346845
1.02	6	3.9619256	3.96192477
1.06	6	4.1043170	4.10431753
1.10	6	4.1958467	4.19584621
1.20	6	4.2103399	4.21033893
1.22	6	4.1785340	4.17853304
1.26	6	4.0829542	4.08295339
1.30	6	3.9475287	3.94752847
1.32	6	3.8660082	3.86600921

x	n	*for*	*read*
1.34	6	3.7759349	3.77593384
1.40	6	3.4595341	3.45953334
1.42	6	3.3404608	3.34046151
1.46	6	3.0852220	3.08522283
1.52	6	2.6680571	2.66805791
1.54	5	0.5802503	0.58025050
1.60	6	2.0712481	2.07124870
1.74	6	1.0095367	1.00953632
1.76	5	0.9520677	0.95206724
1.78	5	0.9679026	0.96790228
1.80	4	−0.4691537	−0.46915342
1.80	6	0.5801438	0.58014344
1.82	5	0.9911308	0.99113044
1.88	5	1.0059216	1.00592110
1.90	3	−0.0760477	−0.07604872
1.92	4	−0.3491837	−0.34918347
1.92	5	1.0033660	1.00336536
1.94	5	0.9986367	0.99863612
1.96	5	0.9917359	0.99173665
1.98	5	0.9827682	0.98276891
2.00	5	0.9718380	0.97183739
2.06	4	−0.2128736	−0.21287344
2.06	5	0.9283347	0.92833416
2.06	6	−0.8480016	−0.84800114
2.08	6	−0.9216997	−0.92169927
2.10	6	−0.9898676	−0.98986749
2.12	6	−1.0525184	−1.05251861
2.14	3	−0.1365916	−0.13659143
2.14	5	0.8494399	0.84943889
2.14	6	−1.1096856	−1.10968436
2.16	5	0.8267198	0.82671889
2.16	6	−1.1614158	−1.16141445
2.18	6	−1.2077754	−1.20777570
2.22	6	−1.2847396	−1.28473822
2.24	5	0.7270882	0.72708742
2.24	6	−1.3155509	−1.31554946
2.26	5	0.7005090	0.70050969
2.26	6	−1.3414083	−1.34140970
2.28	6	−1.3624566	−1.36245589
2.30	3	−0.1491983	−0.14919850
2.30	4	−0.0214149	−0.02141240
2.30	5	0.6460417	0.64604256
2.30	6	−1.3788340	−1.37883586
2.34	5	0.5904438	0.59044322
2.34	6	−1.3982379	−1.39823660
2.36	5	0.5624388	0.56243808
2.36	6	−1.4015997	−1.40159796
2.38	5	0.5342418	0.53440589
2.38	6	−1.4009731	−1.40097219
2.40	5	0.5064238	0.50642453
2.40	6	−1.3965440	−1.39654584
2.42	6	−1.3885091	−1.38851010
2.44	5	0.4509079	0.45090689
2.44	6	−1.3770629	−1.37705991
2.46	5	0.4235077	0.42350716
2.46	6	−1.3623946	−1.36239298
2.48	6	−1.3447081	−1.34470892
2.54	6	−1.2755618	−1.27556009
2.56	5	0.2924724	0.29247276

x	n	*for*	*read*
2.56	6	−1.2478101	−1.24781146
2.58	6	−1.2180421	−1.21804283
2.60	5	0.2437638	0.24376322
2.60	6	−1.1864509	−1.18644823
2.62	6	−1.1532160	−1.15321833
2.64	6	−1.1185374	−1.11853985
2.66	5	0.1756303	0.17563083
2.66	6	−1.0825918	−1.08259508
2.68	3	−0.1232624	−0.12326281
2.68	5	0.1543471	0.15434760
2.68	6	−1.0455579	−1.04556138
2.70	3	−0.1207053	−0.12070568
2.70	6	−1.0076074	−1.00761072
2.72	5	0.1140524	0.11404817
2.84	6	−0.7294393	−0.72943953

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196.—J. PETERS, *Eight-Place Table of Trigonometric Functions for every Sexagesimal Second of the Quadrant.* Berlin, 1939; Ann Arbor, 1943. p. 585, cos 29°10′55″ *for* 87397577 *read* 87307577.

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197.—U. S. HYDROGRAPHIC OFFICE, *Tables of Computed Altitude and Azimuth.* Publication 214.

During the course of a recent computation project at the U. S. Naval Observatory it was found advisable to put on punched cards approximately one-third of the tabular functions contained in H.O. 214, Tables of Computed Altitude and Azimuth, volumes 1–9. The functions for latitudes 30°–39° were taken from a manuscript, kindly furnished by the Hydrographic Office, which had been used in correcting the grosser errors in the first computation of volume 4. This made possible a critical examination of these data by differencing methods designed to detect minimum errors of 0′.2 in altitude and 0°.2 in azimuth, that is, errors of two units or more in the last printed place.

Of the eighteen errata listed below only one might seem to qualify as a computational error, the others apparently being typographical faults. Even if all eighteen were considered to be computational errors, they would amount to only seven thousandths of one percent of the total number of functions examined.

This is the most extensive study of the accuracy of H.O. 214 that has yet been made. Earlier conclusions [*MTAC*, v. 1, p. 81, v. 2, p. 182, v. 3, p. 139, 315] were based on relatively small samples. If the data examined are representative of the data not examined (a reasonable assumption) it may be concluded that the number of errors of two units or greater in H.O. 214 average six per volume, which is a high standard of accuracy.

Errata:

V. 1; p. 75, lat 3°, dec 13°, h.a. 82°, *for* alt 7°00′.2 *read* 7°06′.2.
p. 131, lat 5°, dec 21°, h.a. 0°, *for* alt 64°30′.0 *read* 64°00′.0.
V. 2; p. 110, lat 14°, dec 11°, h.a. 23°, *for* alt 67°23′.1 *read* 67°21′.3.
p. 186, lat 17°, dec 7°, h.a. 80°, *for* az 85°.1 *read* 86°.1.
V. 3; p. 31, lat 21°, dec 7°, h.a. 20°, *for* alt 55°77′.9 *read* 55°47′.9.
p. 112, lat 24°, dec 12°, h.a. 44°, in alt 46°39′.9 the 6 is poorly printed.
V. 5; p. 55, lat 42°, dec 1°, h.a. 31°, in alt 38°41′.9 the 9 is poorly printed.
p. 120, lat 44°, left hand h.a., *for* first 13° *read* 12°.
p. 121, lat 44°, dec 30°, h.a. 31°, *for* az 150°.0 *read* 153°.0.
p. 141, lat 45°, dec 17°, h.a. 14°, *for* alt 26°43′.2 *read* 26°42′.3.
V. 6; p. 7, lat 50°, dec 9°, h.a. 51°, *for* alt 16°14′.7 *read* 16°14′.5.
p. 58, lat 52°, dec 10°, h.a. 88°, *for* az 95°.4 *read* 85°.4.
V. 7; p. 41, lat 61°, left hand h.a., *for* second 132° *read* 133°.
p. 147, lat 65°, dec 30°, h.a. 146°, *for* alt 8°26′.7 *read* 8°36′.7.
V. 8; p. 244, lat 79°, dec 18°, h.a. 49°, *for* alt 24°95′.2 *read* 24°59′.2.
V. 9; p. 37, lat 81°, dec 18°, h.a. 160°, *for* alt 9°3′.13 *read* 9°31′.3.
p. 162, lat 86°, dec 11°, h.a. 62°, *for* alt 12°5′.13 *read* 12°51′.3.
p. 217, lat 88°, left hand h.a., *for* second 132° *read* 133°.

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UNPUBLISHED MATHEMATICAL TABLES

131[E, L].—R. M. COGHLAN & R. C. T. SMITH, *Table of roots of* $\sin z = kz$. Typewritten Manuscript, 2 leaves, on deposit in UMT FILE and with Aeronautical Research Laboratories of the Department of Supply, Box 4331 G.P.O. Melbourne, Australia.

The table gives 6D values of the real and imaginary parts of the first 11 zeros of $\sin z + kz$ for $\pm k = 0(.25)1$. [The results for $k = 1$ have been published; see *MTAC*, v. 3, p. 414, RMT 611.]

132[F].—R. A. LIENARD, *Tables of the factors of* $2^n - n - 2$ *and* $2^n - n - 3$ for $n = 1(1)1000$.

These two functions of n are remarkable in that they do not seem to represent primes. In fact 3 is the only prime that they are known to represent. Most values have small prime factors. In this respect these functions resemble CULLEN's[1] function $1 + x2^x$. At least one prime factor of $2^n - n - 2$ is given for $n \leq 1000$ except for the 17 values: $n = 253, 323, 355, 455, 493, 497, 517, 535, 559, 589, 649, 713, 749, 815, 895, 901, 979$. At least one prime factor of $2^n - n - 3$ is given for $n \leq 1000$ except for the 12 values: $n = 162, 210, 254, 320, 330, 416, 590, 650, 738, 780, 872, 914$.

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[1] A. J. C. CUNNINGHAM & H. J. WOODALL, "Factorisation of $Q = (2^q \mp q)$ and $(q \cdot 2^q \mp 1)$," *Messenger Math.*, v. 47, 1917, p. 1–38.

133[F].—F. L. MIKSA, *Table of primitive Pythagorean triangles with their perimeters arranged in ascending order from* 119992 *to* 499998. 506 typewritten leaves on deposit in UMT FILE.

This table is a continuation of UMT 111 [*MTAC*, v. 5, p. 28], a table by A. S. ANEMA to 120000. The introduction gives a table showing the number of triangles whose perimeters do not exceed P for $P = 120000$ (10000) 500000. The total number of these triangles is 35114. LEHMER'S asymptotic formula gives 35115. Similar data are given for pairs of triangles having equal perimeters of which there are 1750. There are 65 cases of 3, and one case of 4 isoperimetric triangles.

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134[L].—Y. L. LUKE & D. UFFORD, *Tables of* $\int_0^\infty (zt)^{-1}(z + t - \sqrt{z^2 + t^2})e^{-it}dt$. 8 mimeographed leaves on deposit in UMT FILE and also available from Midwest Research Institute, Kansas City 2, Missouri.

The tables give the real and imaginary parts $U + iV$ of the integral given in the title together with the function

$$U + \log 2z + \gamma - 1.$$

6D values of the three functions are given for $z = 0(.01).1(.1)4(.2)5$.

135[L].—J. E. WILKINS JR. & NINA KROPOFF, *Table of Laguerre Functions*. Seven mimeographed leaves on deposit in the UMT FILE.

The table gives 4D values of

$$L_n(x)/n! = M(-n, 1, x) = \sum_{k=0}^{n} (-x)^k \binom{n}{k}/k!$$

for $n = 2(1)7$ and $x = 0(.1)10(.2)20$. [See *MTAC*, v. 1, p. 361, 425, v. 2, p. 31, 267.]

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AUTOMATIC COMPUTING MACHINERY

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TECHNICAL DEVELOPMENTS

Provision for Expansion in the SEAC

In developing the SEAC, two divergent objectives had to be attained. The first objective was to get a modest performance high-speed computer into operation at the earliest possible date; the second objective was to

develop an effective experimental tool for evaluating advanced computer components and systems. In order to reconcile these conflicting aims, it was decided that the SEAC should be made *expandable*, i.e., should consist of a central nucleus (with a high priority on early completion) to which additional units could be added later, as time permitted or as initial operating experience suggested.

The central nucleus of the SEAC was completed and put into regular operation in the Spring of 1950. Since then, several additional units have been incorporated into the machine's memory, input-output, and control equipment.

1. *Memory.*—The first major change made in the SEAC was to double the size of the high-speed internal memory. Initially the SEAC was operated with an acoustic delay-line memory holding 512 words stored serially in 64 mercury tanks (with average access time: 168 microseconds). Subsequently an additional high-speed memory was incorporated which holds 512 words stored in 45 Williams' tubes. All 45 tubes simultaneously supply the computer with 45 binary digits in parallel (with average access time: 12 microseconds). Although the SEAC is in other respects a serial machine, provision was made in the original design for the use of both serial and parallel types of memory. Conversion of the word-form back and forth from serial to parallel is accomplished by means of a shift register with a special flexible control.

2. *Input-Output.*—The same shift register equipment is sufficiently versatile to allow the SEAC to operate with completely asynchronous, serial or parallel, input-output equipment. For example, the SEAC has been operated in conjunction with Teletype punched-paper tape, with Teletype keyboard and printer, with a visual (cathode-ray tube) high-speed graph plotter, with single-channel magnetic wire, and with single-channel magnetic tapes. Additional units which are planned to be incorporated in the near future include multi-channel magnetic tapes, Flexowriter punched-paper tape and printing equipment, punched-card-to-magnetic tape conversion equipment, and a high-speed photographic printer. Also, an external selector is being added to the SEAC which will be capable of selecting (under the computer's control) from among any of ten different input-output units.

3. *Control.*—Another major change to be made in the SEAC will be the addition of equipment to the control. Provision was made for this expansion in the original design of the machine, and, as a result, the operator of the computer will be able to choose (before entering his problem) between two distinct modes of operation (i.e., between the 4-address or the 3-address mode) merely by throwing a few toggle switches.

The main distinction between the 4-address and the 3-address mode is indicated in Table I, which lists the information contained in an instruction-word in each system. Note that the first three items of information are the same in both systems. These items indicate where in the memory the operands are located (α and β), and also where in the memory the result of a given operation on them is to be sent (γ). In the 4-address system, the instruction also contains an explicit statement on where the *next* instruction is located in the memory (δ). In the 3-address system, however, there is no such explicit statement. Instead, instructions in this system are automati-

cally sequenced, i.e., successive instructions being located, ordinarily, in successively-numbered memory positions.

The types of operations which can be performed in both the 3-address and 4-address systems, are similar. They include (besides input-output operations) addition, subtraction, multiplication, division, and a type of extract operation called "logical transfer." Also, three types of comparison operations (sometimes called discrimination or branch operations) can be performed. In a typical comparison operation, when the result of the comparison between two operands indicates that the first operand is greater than or equal to the second operand, then the next instruction is chosen normally, in the manner just described. *But*, if the second operand is the greater, then the next instruction in both systems is chosen from address γ.

4. *Floating Address Feature.*—In the 3-address system, a special automatic feature is included which is believed to be novel and which is referred to as the Floating Address feature. This was incorporated into SEAC in order to shorten and simplify the work of the programmer. It does not accomplish any effects which the human programmer could not also accomplish at the cost of additional labor. The scheme does, however, automatize certain time-consuming programming operations, thereby completely relieving the programmer of any need for concerning himself with them. This is accomplished at the cost of a relatively small amount of additional equipment.

In brief, this feature makes it easy for the programmer to reutilize standard routines or lists of instructions which have already been compiled for carrying out certain frequently-used operations, such as taking a square root, making a binary-to-decimal conversion, and so on. Obviously he would like to make full use of any such material originally prepared for one problem in the solution of other problems. But there is a difficulty in the way of such straightforward re-use of standard routines. This arises from the fact that the *memory locations* into which he must insert the standard list of instructions will be different in general for each new problem. This difference forces him to change the memory address by which he identifies each individual instruction in the routine each time he uses the routine in a different problem. The *floating address* feature performs this change of address *automatically* each time the routine is used, and in a manner which requires no attention on the part of the programmer.

Table II illustrates the characteristics of the floating address feature. In preparing a routine originally, the programmer can designate any address as either *absolute* or *relative*. An absolute address is interpreted in the usual sense merely as a number identifying a specific memory location. A relative address, however, identifies a position in the memory by specifying its displacement relative to the position of the instruction-word itself. To illustrate how this works: if an instruction located in memory position m specifies that the address of the first operand is number 17 *relative*, then the control will take the word in memory location $m + 17$ and use it as the first operand. To refer to location $m - 17$, the programmer uses as address the complement of 17 ($2^{12} - 17$, relative). It is possible to use any combination of absolute or relative addresses in any instruction word.

By utilizing this feature, therefore (by using relative-address designations for all references to instructions inside the routine itself), routines may be prepared in advance once and for all, transcribed on tape, and

stored away in the library files. For any subsequent problem, then, each routine can be inserted in the memory in any arbitrary location whatever, and executed immediately without need for any preparatory modification to adapt it to the particular memory location in which it happens to fall.

The second major characteristic of the floating address feature is intended to relieve the programmer of another related type of burden. To illustrate this, let us represent the various storage locations of the internal memory by the boxes shown on the flow chart (Table III). Each box contains an instruction. Each group of boxes represents a typical routine, a list of instructions stored in consecutively numbered address locations. The right-hand list represents a sub-routine describing some standard process which has been entered into the memory verbatim from a strip of library tape. The left-hand list represents the main routine prepared specially for the problem at hand. During the course of any problem, the control usually follows the main routine for a while, then jumps to some sub-routine, then jumps back to the main routine, and so forth. Now the same sub-routine generally will be referred to repeatedly during the course of a problem. (E.g., square roots may be sprinkled in many places throughout the computation.) The point of departure from the main routine will in general be different in each case. Under the 4-address SEAC system, the programmer has to arrange, before embarking on the sub-routine, to plant some sort of clue in some pre-arranged location (or at the end of the sub-routine itself) which will lead the control back to the main routine at the proper point of return each time. This means programming extra operations.

With the 3-address floating address system, the return to the proper point in the main routine always takes place automatically and requires no attention at all from the programmer. This is accomplished in the following manner: The system provides two small storage registers, each of which is capable of holding an address number. Normally, as soon as any instruction is executed, the contents of *one* of these registers are advanced one unit, and the number it contains is taken as the address of the next instruction. The contents of the other register are held unchanged. A single binary digit (called "d" on Table II) is reserved in each instruction word for designating which one of the two registers is to be referred to.

To be more specific, on main routines the instructions are normally written with the register designation digit $d = 0$. These main routines are sequenced by the counter register called #0. To embark on a sub-routine, the programmer writes a special "jump" instruction (with $d = 1$) which causes the other counter register (called #1) to take over. The sub-routine, always written with the digit $d = 1$, continues using counter #1. The *final* instruction of the sub-routine, however, is written with digit $d = 0$. Immediately after executing the final instruction, therefore, counter #0 is referred to for the address of the next instruction, and the control is back on the main routine at the point immediately following where it left off. All sub-routines are written in an invariant form in this respect, and no other attention on the part of the programmer is required.

Some of the alternative courses followed by the control are indicated in Table III: *normal* operation, *jump* operation, and the path of *automatic return*. By combining this dual-counter feature with the relative address feature, a number of other useful operations may be performed. As an

example of one: by a jump operation with a relative resetting for counter #0 at the end of the sub-routine, automatic return may be made to take place to any part of the main routine.

Now the question naturally arises: How much does this added control equipment increase the size of the computer? The answer in terms of the total tube count of the complete computer is: about 3% (30 tubes out of a total of about 1050). Since the SEAC was originally designed with the possibility of these additions in mind, the actual process of attaching new units is quite simple and straightforward.

Is annexation of such additional equipment justified? We believe it is. The value of the computer ought to be measured by its total productivity in terms of complete problems solved. Experience to date with the SEAC suggests that in the long run the main bottleneck on problem output may be the ability of the human operator to prepare and check adequately the programs that he enters into the machine. Preparing a program for a large scale problem is a highly intricate process and demands complete accuracy of detail. Furthermore, checking a program which appears to be going astray on the machine often turns out to be very time-consuming. It is found, for example, that the time necessary for locating and correcting *program* errors is often comparable with the time spent in locating and correcting *machine* errors. With the advent of (first) more complicated problems and (second) more reliable machines, the balance will become even more heavily weighted against the programmer. By simplifying his job and reducing the number of details with which he has to concern himself, we think we can reduce the incidence of human error and thereby gain markedly in efficiency. For this reason, we feel that the supplementary equipment just described will contribute substantially towards increasing the SEAC's problem-solving productivity.

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TABLE I. THE SEAC'S CONTROL

NOW IN OPERATION	*BEING INCORPORATED*
4-address system	3-address system (with Floating Address feature)
Each instruction word contains:	Each instruction word contains:
(1) Address of first operand (α) ... 10 binary digits	(1) Address of first operand (α) ... 12 binary digits
(2) Address of second operand (β) ... 10 binary digits	(2) Address of second operand (β) ... 12 binary digits
(3) Address of result (γ) ... 10 binary digits	(3) Address of result (γ) ... 12 binary digits
(4) Address of next instruction (δ) ... 10 binary digits	(4) Floating address information (a, b, c, d) ..... 4 binary digits
(5) Symbol specifying type of operation to be performed .... 4 binary digits	(5) Symbol specifying type of operation to be performed .... 4 binary digits
(6) Halt signal .... 1 binary digit	(6) Halt signal .... 1 binary digit
Total: 45 binary digits in each word	Total: 45 binary digits in each word

TABLE II. FLOATING ADDRESS INFORMATION

RELATIVE ADDRESS INFORMATION

(a) indicates whether α is an absolute or relative address .... 1 binary digit

(b) indicates whether β is an absolute or relative address .... 1 binary digit

(c) indicates whether γ is an absolute or relative address .... 1 binary digit

DUAL REGISTER INFORMATION

(d) modifies choice of next instruction ..... 1 binary digit

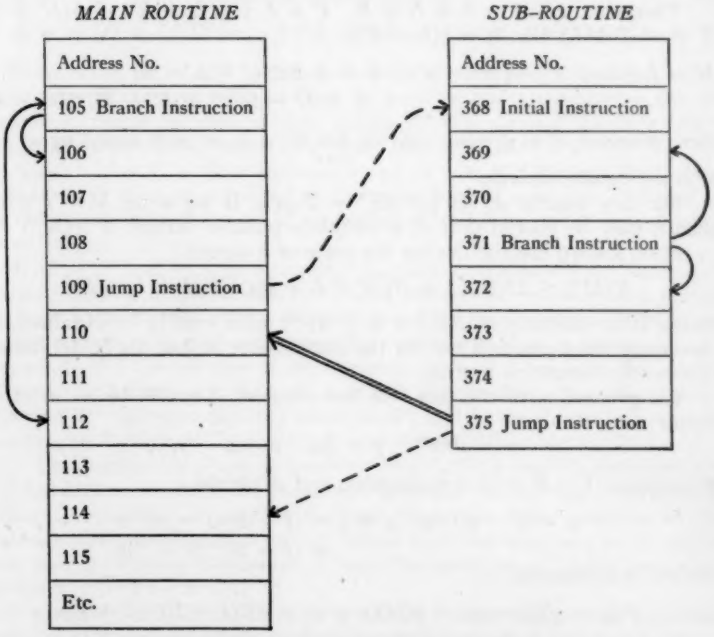
Total: ..... 4 binary digits

TABLE III. CHOICE OF NEXT INSTRUCTION IN THE THREE-ADDRESS (FLOATING) SYSTEM

POSSIBILITIES JUMP operation, indicated by: - - - →

NORMAL operation, indicated by: ⟶

PATH OF AUTOMATIC RETURN, indicated by: ⟹



Notes on Numerical Analysis—6

The Convergence of Seidel Iterants of Nearly Symmetric Matrices

If A is a matrix, we may write $A = D + L + R$. Here D is the matrix of its leading diagonal elements, L the triangular matrix of the elements

below this diagonal, and R the triangular matrix of the elements above. If B is the Seidel[1] iterant of A, then $B = -(D+L)^{-1}R$.

It is known that if A is symmetric, $\lim_{n\to\infty} B^n = 0$ if A is positive definite.[2]

By continuity one may infer that if A is 'nearly equal' to some symmetric matrix C, the Seidel iterants of A and C would behave in a similar way with regard to convergence. For, although there is a limiting process involved, the convergence of B^n to 0 depends on the maximum modulus of the characteristic roots of B, and this is a continuous function of the elements of B. The problem attempted in this paper is to determine a precise meaning for the notion 'nearly equal.'

Any matrix A can be written in the form $A = C + S$, where C is symmetric and $S = \lambda - \lambda'$ is skew-symmetric (λ is defined in the Theorem). C may be regarded as the symmetric matrix nearest to A, and λ is then a measure of its asymmetry. Our solution of what is to be understood by 'nearly equal' is given in terms of C and λ. The result is simplest when $D = I$, the unit matrix. We state the result in that case as a theorem. The minor adjustments required when $D \neq I$, all $a_{ii} > 0$, are explained at the end of the paper.

Theorem. *If* $A = I + L + R$, $C = I + \frac{1}{2}(L + R') + \frac{1}{2}(L' + R)$, $B = -(I+L)^{-1}R$, $\lambda = \frac{1}{2}(L - R')$,

$$M = I + \lambda + \lambda' - \tfrac{1}{2}\{(\lambda - \lambda')(I + L + R)^{-1}(I + L - R) + (I + L' - R')(I + L' + R')^{-1}(\lambda' - \lambda)\},$$

then, provided M *is positive definite,* $\lim_{n\to\infty} B^n = 0$, *or* $\neq 0$ *according as* C *is or is not positive definite.*

For any matrix A, let $[N(A)]^2 = \Sigma_{i,j} a_{ij}^2$. If we write $M = I + M^*$, then it may be proved that M is certainly positive definite if $N(M^*) < 1$.

From known inequalities for the norm of a matrix,[3]

$$N(M^*) \leq 2N(\lambda)\{1 + N[(I + L + R)^{-1}]N(I + L + R)\}.$$

Hence if an estimate of $N\{(I + L + R)^{-1}\}$ may readily be obtained, the theorem gives a practical test for the convergence to 0 of the Seidel iterants of a nearly symmetric matrix.

We proceed with the proof of the theorem. Let $x^{(0)}$ be an arbitrary vector; let

$$x = B^n x^{(0)}, \quad y = Bx, \quad z = x - y.$$

We suppose $I - B$ to be non-singular, and so obtain

$$x = (I - B)^{-1}z, \quad x + y = (I + B)(I - B)^{-1}z = (I - B)^{-1}(I + B)z.$$

Since C is symmetric,

$$(1) \qquad x'Cx - y'Cy = (x' - y')C(x + y) = z'C(I - B)^{-1}(I + B)z.$$

Since $B = -(I + L)^{-1}R$, and $C = A + \lambda' - \lambda$, substituting in (1) we get

$$(2) \qquad x'Cx - y'Cy = z'(I + L + R - \lambda + \lambda')(I + L + R)^{-1}(I + L - R)z.$$

We interrupt the proof to state a lemma:

Lemma. *If C is a symmetric matrix, $y = Bx$, $x'Cx - y'Cy > 0$, then* $\lim_{n\to\infty} B^n = 0$ *or* $\neq 0$, *according as C is positive definite or not.*

A formal proof of the lemma is given elsewhere.[4]

If now $\lambda = 0$ then (2) gives

$$x'Cx - y'Cy = z'z > 0 \text{ if } z \neq 0.$$

Since $z = (I - B)x$ and $I - B$ is not singular, $z \neq 0$ if $x \neq 0$. By the lemma the known result for the convergence of the Seidel iterants then follows immediately.

If A is not symmetric, we may continue proving the theorem as follows. For any matrix P, $z'Pz = \frac{1}{2}z'(P + P')z$, and $\frac{1}{2}(P + P')$ is a symmetric matrix. Applying this result to the matrix on the right of (2) we get $x'Cx - y'Cy = z'Mz$, where M is the matrix defined in the enunciation of the theorem. The theorem now follows from the lemma.

If $A = D + L + R$, and $D \neq I$, all $a_{ii} > 0$, the theorem is still true if we generalize somewhat the definitions of C and M. Let δ be the diagonal matrix whose elements are the positive square roots of the elements of D. Then $\delta^2 = D$. The Seidel matrix of A is then

$$B = -(D + L)^{-1}R = -(\delta^2 + L)^{-1}R = -\delta^{-1}(I + \delta^{-1}L\delta^{-1})^{-1}\delta^{-1}R.$$

If we write $B^* = \delta B\delta^{-1}$, the characteristic roots of B^* are those of B. Hence the B iterants will converge to zero or not according as the B^* iterants converge to zero or not. Again $B^* = -(I + \delta^{-1}L\delta^{-1})^{-1}\delta^{-1}R\delta^{-1}$. Hence B^* is the Seidel iteration matrix for $I + \delta^{-1}L\delta^{-1} + \delta^{-1}R\delta^{-1}$.

Now, if $I + L + R$ is symmetric, so is $I + \delta^{-1}L\delta^{-1} + \delta^{-1}R\delta^{-1}$. The theorem may thus be extended to the more general case provided that in the definitions of C, B, λ and M, we replace L and R by $\delta^{-1}L\delta^{-1}$ and $\delta^{-1}R\delta^{-1}$ respectively.

If $A = D + L + R$ is a symmetric matrix, the condition that A is positive definite in order that the Seidel iterants of its associated matrix converge to zero is not only sufficient but, in the case when the elements of D are all positive, is also necessary.[5] For we observe that in the case $D = I$ this result is a special case of the theorem. If $D \neq I$, we form the matrix $A^* = \delta^{-1}A\delta^{-1}$ where δ is defined as above. If now M is any leading minor of A and M^* the corresponding minor of A^*, and δ_m the elements in δ corresponding to M, then $M^* = \delta_m^{-1}M\delta_m^{-1}$ and so $\det(M^*) = (\det \delta_m^{-1})^2 \det(M)$.

If the elements in D are all positive, $(\det \delta_m^{-1})^2$ is positive and $\det(M^*)$ and $\det(M)$ are both of the same sign. Hence A and A^* are both positive definite, or neither is positive definite. Since the Seidel iterants of the matrices corresponding to A and A^* either both converge to zero or neither does, and the leading diagonal in A^* is I, the result for A follows from what has been proved for A^*.

I wish to thank Olga Taussky-Todd for the encouragement I received from her in the course of the preparation of this paper. I also wish to thank my colleague, S. E. Cruise, for reading and checking the manuscript.

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[1] See, for example, HILDA GEIRINGER, "On the solution of systems of linear equations by certain iteration methods," Reissner Anniversary Volume, *Contributions to Applied Mechanics*, Ann Arbor, 1948, p. 365–393.
[2] This is Seidel's result. For a recent proof see R. VON MISES & HILDA GEIRINGER, "Praktische Verfahren der Gleichungsauflösung," *Zeitschr. f. angew. Math. u. Mech.*, v. 9, 1929, p. 58–77, 152–164.
[3] See, for example, p. 1044 in JOHN VON NEUMANN & H. H. GOLDSTINE, "Numerical inverting of matrices of high order," Amer. Math. Soc., *Bull.*, v. 53, 1947, p. 1021–1099.
[4] P. STEIN, "Some general theorems on iterates," to appear in NBS *Jn. of Res.*
[5] A different proof of this result is given by E. REICH, "On the convergence of the classical methods of solving linear simultaneous equations," *Ann. Math. Stat.*, v. 20, 1949, p. 448–451.

BIBLIOGRAPHY Z–XVII

1. ANON., "Conversion between analogue and digital data," ONR London, *Technical Report* ONRL-73-50, July 1950, 4 pages.

This brief report from the Scientific Liaison Officer of the American Embassy in London, England, summarizes a symposium held at the Military College of Science, Shrivenham, 5–6 July, 1950. The following prepared papers were presented and discussed:

(1) "A method of telerotation by pulse code modulation," by R. H. BARKER of the Signals Research and Development Establishment, Christchurch, Hants.,
(2) "The production and use of binary coded discs for data transmission," by W. S. ELLIOTT, R. C. ROBBINS, and D. S. EVANS of the Research Laboratories of Elliott Bros., Borehamwood,
(3) "Telerotation by parallel binary codes," by E. A. JOHNSON of the Radar Research and Development Establishment, Great Malvern, Worcs.,
(4) "A servo system operating on discontinuous information," by C. HOLT-SMITH and D. LAWDEN of the Faculty of Instrument Technology, Military College of Science, Shrivenham,
(5) "Note on digital computing for the extrapolation of discontinuous information," by G. C. TOOTILL of the Faculty of Instrument Technology, Military College of Science, Shrivenham,
(6) "Some methods of conversion of data from analogue (shaft rotation) to digital form," by K. V. DIPROSE of the Royal Aircraft Establishment, Farnborough, Hants.,
(7) "The electromagnetic cam: a technique for wave manipulation using the nonlinear properties of magnetic materials," by W. LAWRENCE of the Signals Research and Development Establishment, Christchurch, Hants.

Copies of these papers have been forwarded to Code 250, Office of Naval Research, Washington 25, D. C., and the proceedings of the Symposium (except for these papers) will be prepared and distributed by the Ministry of Supply.

The different systems described for conversion between shaft rotation and digital codes had different chief features such as compactness, accuracy of one part in 20,000, or a three microsecond digit time. The prediction problem was treated both by direct use of the discrete data and by polynomial approximation to the prediction function.

R. D. ELBOURN

NBSCML

2. ANON., "Desk-sized electronic computer developed by Northrop," *Machinery*, v. 57, Mar. 1951, p. 168.
   Short expository article on the MADDIDA.

3. ANON., *Digital Computer Newsletter*, ONR Mathematical Sciences Division, v. 3, April 1951, 6 pages.
   The contents are as follows:

   1. The ERA 1101 Computer
   2. Moore School Automatic Computer
   3. The Institute for Advanced Study Computer
   4. Aberdeen Proving Ground Computers: The ENIAC, The EDVAC, The ORDVAC
   5. National Bureau of Standards Western Automatic Computer (SWAC)
   6. National Bureau of Standards Eastern Automatic Computer (SEAC)
   7. Project Whirlwind
   8. Relay Digital Computer, Imperial College, Univ. of London
   9. Zuse Computer Model IV, at Zurich, Switzerland
   10. Data Handling and Conversion Equipment: Stavid Engineering Data Conversion Equipment, Signal Corps Angular Position Encoders, and Zatacoding

4. ANON., *Digital Computer Newsletter*, ONR Mathematical Sciences Division, v. 3, July 1951, 6 pages.
   The contents are as follows:

   General Purpose Computers

   1. Burroughs Laboratory Computer
   2. UNIVAC Dedication
   3. The Raytheon Digital Computer
   4. The National Bureau of Standards Eastern Automatic Computer
   5. Whirlwind I (M.I.T.)
   6. The ORDVAC (Univ. of Ill.)
   7. Moore School Automatic Computer (MSAC)
   8. The Institute for Advanced Study Computer

   Special Purpose Computers

   1. The CRC 101 Digital Differential Analyzer
   2. The MADDIDA

   Auxiliary and Conversion Equipment

   1. Flying Typewriter
   2. Conversion and Display Equipment

5. ANON., "Guided-missile computer," *Electronics*, v. 24, Apr. 1951, p. 138.
   This is an expository paper which describes the hybrid analog-digital computer being developed at Project Typhoon, Special Devices Center of ONR, Port Washington, N. Y.

6. T. J. CONNOLLY, S. P. FRANKEL, & B. H. SAGE, "Application of automatic digital computing methods to the production of phase behavior," *Electrical Engineering*, v. 70, Jan. 1951, p. 47.

Punched card methods using the type 604 calculating punch were developed to predict the thermodynamic properties of homogeneous and heterogeneous multicomponent hydrocarbon systems by solving the Benedict equation of state. Methods described permit evaluating the composition and the molal values of volume, entropy, and enthalpy of the coexisting phases at one heterogeneous state in about one hour of computing, or for a single homogeneous phase in about ten minutes. Thus computing particular values as required is feasible whereas tabulating completely the thermodynamic properties of multicomponent heterogeneous systems is not. A bibliography of 36 titles is included.

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NBSCML

7. D. H. GRIDLEY & B. L. SARAHAN, "Design of the Naval Research Laboratory Computer," *Electrical Engineering*, v. 70, Feb. 1951, p. 111.

It is expected that the Naval Research Laboratory Electronic Digital Computer (NAREC) will be working by January 1952 and will be in productive operation within six months following. It will have 1024 words of electrostatic storage and 2048 of magnetic drum storage. Words will consist of 44 binary digits and sign; instruction words will be interpreted as pairs of one-address orders. Input and output will be on magnetic tape. The machine will have a multiplication time of 300 microseconds.

MICHAEL MONTALBANO

NBSCL

8. LAWRENCE P. LESSING, "The electronics era," *Fortune*, v. 44, July 1951, p. 79–83, 132–138.

Briefly mentioned in this article are the important recent developments in the computer field. To quote from the paper: "It may be that the most significant accomplishment for electronics for the future is its ability to count."

9. D. M. MCCALLUM & J. B. SMITH, "Mechanized reasoning, logical computers and their design," *Electronic Engineering*, v. 23, Apr. 1951, p. 126–133.

A small electrical computer, built at Ferranti, Ltd. in Edinburgh, to evaluate Boolean propositional functions of up to seven binary-valued variables is described. A function is set up by plugging together logical connective boxes which contain relays, selenium diodes, and indicator lamps. The computer then tries automatically in sequence the 128 combinations of values of the variables and stops on any combination for which the function is "true."

One possible extension described is particularly interesting because a computer using it might find reasonably quickly a solution for a function of so many variables that systematic scanning would take a prohibitively long time; moreover, the process appears to be more like that used by the

human brain. The method is to assume some set of values of the variables, which usually will not be a solution, then to identify those variables whose values appear to be responsible for the false result and, by feedback, to cause them to change more or less at random until a true result is achieved. A scheme for avoiding circular sequences of changes is suggested.

NBSCML R. D. ELBOURN

10. AMBROS P. SPEISER, *Entwurf eines Elektronischen Rechengerates*, Inst. f. angew. Math., *Mitt.*, Zürich, No. 1, 1950.

This is a description of the logical design of a digital computer intended to be built at low cost and to operate at relatively low speed, but otherwise to equal the performance of most of the modern automatic machines. No new engineering development is contemplated; use is made only of existing and well-tested components, especially of those developed by H. H. Aiken. The memory organ is a magnetic drum holding 1200 words, each consisting of sign and 12 decimal digits. The average access time is 16 milliseconds. The arithmetic organ operates serially, with a pulse repetition rate of 40 k.c. The addition time is .8 ms; the average multiplication time 30 ms. Multiplication is accomplished by repeated addition, division by an original modification of the repeated subtraction process. Instructions are written in a single-address system, pairs of addresses being stored in the memory as single words. The arithmetic unit and part of the control are electronic, but address selection is performed by relays. The design calls for 1000 vacuum tubes and 500 relays in the entire machine. Teletype equipment is used for input and output, without provision for external storage. There is no automatic checking, except that the machine stops when certain "forbidden" pulse combinations occur.

NBSCL FRANZ L. ALT

11. H. W. SPENCE, "Systematization of tube surveillance in large scale computers," *Electrical Engineering*, v. 70, July 1951, p. 605–608.

Ninety per cent of the service interruptions of ENIAC at Aberdeen Proving Ground are caused by vacuum-tube failures. This paper describes the first five months' operation of a new tube-surveillance program. The average life of some 5000 tubes discarded varied among ten types from 5,500 to 12,500 hours. Much of the difference between types is attributed to different operating and testing conditions. For most types low emission is the predominant cause of failure, especially so in normally OFF tubes. If only low-emission failures are considered, the life of tubes used in the ON condition is better than that of the same type used in the OFF condition by 50 per cent or more. Poor cutoff, short circuited elements, and burned out filaments may be considered accidental failures because life terminated by these averaged less than half as long as that for low-emission failure.

Because it contains the first extensive tube life data appropriate to computer design, this report is especially welcome and further results are eagerly awaited.

NBSCML R. D. ELBOURN

News

Department of the Air Force and National Bureau of Standards.—A symposium on Linear Inequalities and Programming was held in Washington, D. C., on June 14, 15, and 16, 1951. This meeting was sponsored by the Department of the Air Force and the National Bureau of Standards. The program was as follows:

Thursday, June 14, Morning	
Linear inequalities	J. Todd, *Chairman*, USAF
Welcoming talk	Gen. F. J. Dau, USAF
Theorems of alternatives for pairs of matrices	A. W. Tucker, Princeton Univ.
Remarks on the history of work on linear inequalities	T. S. Motzkin, NBSINA
Optimization in unitary spaces	E. W. Barankin, Univ. of Calif. and Rand Corp.
Polyhedral cones	A. Charnes, Carnegie Inst. of Tech.
Geometrical significance of the simplex method	J. P. Mayberry, Princeton Univ.
Discussion	
Thursday, June 14, Afternoon	
Applications	T. C. Koopmans, *Chairman*, Cowles Commission, Univ. of Chicago
Choice models and non-choice models	W. W. Leontief, Harvard Univ.
Research program of Project SCOOP	M. K. Wood, USAF
Discussion	
Friday, June 15, Morning	
Computational theory and techniques	G. B. Dantzig, *Chairman*, NBSCL
Welcoming talk	Gen. F. J. Dau, USAF
New techniques for linear inequalities and optimization	T. S. Motzkin, NBSINA
Methods of solving linear equations	G. Forsythe, NBSINA
Gradient methods in Lagrangian problems and their game theoretical interpretation	L. Hurwicz, Univ. of Illinois, and Cowles Commission, Univ. of Chicago
Properties of Leontief matrices	Y. K. Wong, Princeton Univ.
Problems of formulation	P. A. Samuelson, *Chairman*, MIT and Rand Corp.
Efficiency prices for decentralized decisions	T. C. Koopmans and G. Debreu, Cowles Commission, Univ. of Chicago
Suitability scales for allocation problems	J. L. Holley, USAF
Personnel classification	D. F. Votaw, Jr., Yale Univ., and A. Orden, USAF
Design of an optimal battery of tests	M. A. Woodbury, Princeton Univ.
Saturday, June 16, Morning	
Nonlinear and linear problems	E. D. Schell, *Chairman*, USAF
Theory of the transportation problem	M. M. Flood, Rand Corp.
Convex programming	D. W. Blackett, Princeton Univ.
Least ballast shipping required to meet a specified shipping program	I. Heller, Logistics Research, George Washington Univ.
Problem of contract awards	L. Goldstein, USAF
A gasoline blending problem	W. Cooper, Carnegie Inst. of Tech.
Discussion	

Saturday, June 16, Afternoon

Computational theory and techniques	F. L. ALT, *Chairman*, NBSCL
Matrix inversion	H. H. GOLDSTINE, Inst. for Advanced Study, Princeton Univ.
A unified technique for matrix inversion, linear inequalities, games, optimization	A. ORDEN, USAF
A short proof of the dual theorem	G. B. DANTZIG, USAF
The projective method and computational experience	C. B. TOMPKINS, Logistics Research, George Washington Univ.
Discussion	

Eckert-Mauchly Division, Remington Rand Inc.—The first UNIVAC, which was constructed for the Bureau of the Census, under NBS contract with Remington Rand, was delivered by EDWARD U. CONDON, Director of the NBS, to ROY V. PEEL, Director of the Bureau of the Census, on June 14th.

During the elaborate ceremonies which took place in Philadelphia on that occasion, the UNIVAC was busy on a typical task of the Bureau of the Census—the tabulation of information about JOHN Q. CITIZEN. In one-sixth of a second, the machine classified an individual according to place of residence, sex, color, age group, nationality, citizenship, extent of education, educational activity, farming activity, government activity, marital status, domestic status, veteran status, employment status, occupational group, industrial group, earning capacity, etc. For each county, the resulting tabulations were printed out by the UNITYPER under the appropriate headings. Any interested guest at the dedication ceremonies had the opportunity to check the crossfootings testing the accuracy of the machine's operations.

Even at the amazingly rapid rate of six persons per second, the classification of 150,000,000 individuals and the preparation of the various and sundry tabulations is too formidable a task for a single machine. It is planned therefore to continue the use of punched card equipment as heretofore, in conjunction with the tabulation by the UNIVAC of the residents of a few selected states. The Bureau of the Census estimates that about ten per cent of the total task will be accomplished by electronic equipment by the time the 1950 tabulations will have been completed.

During the morning of June 14th, a press seminar was held at which the following program was presented:

Welcome	ALBERT GREENFIELD, President, Chamber of Commerce, Philadelphia
Description of the UNIVAC	J. PRESPER ECKERT, Eckert-Mauchly Division, Remington Rand, Inc.
Description of Demonstration Problem	JAMES L. MCPHERSON, Bureau of the Census
Demonstration of the UNIVAC	S. N. ALEXANDER, NBS
Press Forum	

During the afternoon the dedication ceremonies were held. The program was as follows:

Chairman	ALBERT GREENFIELD, President, Chamber of Commerce, Philadelphia
Economic Implications	JAMES H. RAND, President, Remington Rand, Inc.
Official Delivery of the UNIVAC	EDWARD U. CONDON, Director, NBS, and ROY V. PEEL, Director, Bureau of the Census
Dedication	CHARLES SAWYER, Secretary of Commerce

OTHER AIDS TO COMPUTATION

A Graphic Solution of Simultaneous Linear Equations

This note presents a graphic method of solving sets of simultaneous linear equations which can easily be extended to the inversion of matrices and the calculation of determinants. Its main advantages are speed and simplicity, its main disadvantage is the approximate nature of graphic solutions.

The procedure to be followed involves the repeated use of the following diagram:

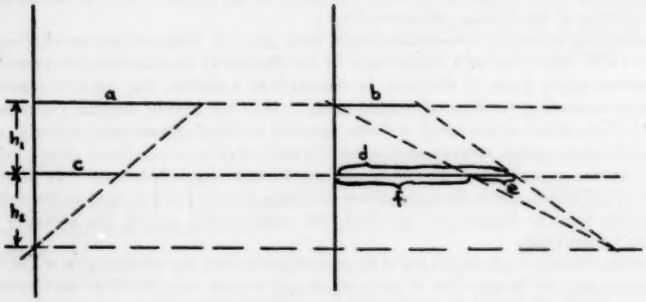

Figure 1

The dotted lines are drawn parallel, the vertical distance between the top two being h_1, between the bottom two being h_2. Since c is parallel to a and since e is parallel to b, the ratios c/a and e/b are each proportional to $h_2/(h_1 + h_2)$. That is,

$$c/a = h_2/(h_1 + h_2) = e/b.$$

Therefore, $e = cb/a$, and

$$f = d - e = d - cb/a. \tag{1}$$

Now, if the given equations are

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= a_{10} \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= a_{20} \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n &= a_{n0} \end{aligned} \tag{2}$$

and if Figure 1 is so drawn that the lengths of lines a and b measure a_{11} and a_{1j}, and the lengths of lines c and d measure a_{i1} and a_{ij}, we have

$$f = a_{ij(1)} = a_{ij} - a_{1j}a_{i1}/a_{11} \tag{3}$$

where $a_{ij(1)}$ is the coefficient of x_j in the i-th reduced equation obtained by the elimination of x_1.

The graphic procedure followed here is exactly parallel to the arithmetic of the Gauss method and its numerous variations such as the Doolittle

method. Equations (2) are reduced to

$$
\begin{aligned}
a_{22(1)}x_2 + \cdots + a_{2n(1)}x_n &= a_{20(1)} \\
a_{32(1)}x_2 + \cdots + a_{3n(1)}x_n &= a_{30(1)} \\
\cdot \quad \cdot \quad \cdot \quad \

at n. Fourth, we draw nc, intersecting ij at p. Fifth, we draw fv and extend it to intersect mo at o. Sixth, we draw eo, intersecting kv at q. Now, $a_{22(1)} = ip$, $a_{23(1)} = kq$, and $\beta_{13.2} = kq/ip$.

We can evaluate $\beta_{13.2}$ graphically by drawing the heavy dotted lines below the segments representing the second equation. These lines are located on the 0-line by the intersection of line cn and a line parallel to the 0-line at a distance 1. The value of $\beta_{13.2}$ is measured by line st.

A "back solution" is shown also for $\beta_{12.3}$. First, draw line rd and extend it to meet the 0-line at u. Second, draw uu' horizontal. Third, draw line tf and extend it to intersect uu' at u'. Fourth, draw su', intersecting ef at w. The value of $\beta_{12.3}$ is ew.

Whether the computations are carried out arithmetically or graphically, the number of steps in an n-variable problem increases rapidly as n increases about in proportion to n^3. A diagram for a problem of four or five variables is much larger than Figure 2 and many more points must be located. But, as in the Doolittle method, the entire process is a repetition of the steps taken in Figure 1. And after a little practice it is unnecessary to draw all the lines indicated in Figures 1 and 2. All we need is to locate and mark points of intersection, using a transparent straightedge.

We have found that the reduction of each coefficient takes about five seconds if there are no complications. The complications that arise are caused by a lack of space if two of the initial coefficients are almost equal, causing near parallelism of two lines which must intersect. This can easily be rectified by first going through one of the equations involved and halving each of its coefficients and the constant term.

The scale should be chosen so that there is no length of as long as 8 inches. Generally the result will have accuracy of three significant figures, occasionally less. It should be noticed in the computation that negative coefficients are measured to the left of the 0-line, positive to the right. If more accuracy is needed this process gives an excellent first approximation to use in one of the iterative methods discussed by HOTELLING.[1]

The equipment necessary includes a good drawing board, a T-square and a triangle, together with a rule, preferably transparent with several scales. If one uses colored pencils for consecutive reductions of the set of equations one can do at least a 5×5 set without tracing the partial solution to another sheet.

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[1] HAROLD HOTELLING, "Some new methods in matrix calculation," *Annals Math. Stat.*, v. 14, 1943, p. 1–34.

BIBLIOGRAPHY Z–XVII

12. H. T. AVERY, "The design of office machines—such as calculators," *Mech. Engineering*, v. 71, 1949, p. 719–724. Discussion by W. F. BERCK & J. S. BEGGS, v. 72, p. 344–346.

This paper by the chief engineer of the Marchant Calculating Machine Company discusses the engineering questions associated with the mass production of calculating machines and similar devices and presents an

example of the design of such a calculator. The design example includes a method for introducing an addend continuously by means of gears into the machine. The keyboard selects the proper gear ratio but the actual gear shifting is done automatically. There is also a discussion of a carry system based on differentials, the use of differentials to position the wheels of a register and a continuous method of comparing the remainder in division with the divisor in order to control automatic division.

F. J. M.

13. B. S. Benson, "Description of an automatic data-analyzing machine," *Mech. Engineering*, v. 72, 1950, p. 285–288.

This device has as its input the film record telemetering data from flight tests of airplanes and missiles in bar graph form. In each frame the values of six out of twenty-eight variables are chosen, measured, and plotted. The selection is governed by a counting relay and integrating networks are provided for the measurement by photo cells and calibration of the quantities. The six quantities are distinguished in the graphical recorder by color. The time between frames is used as a reference for checking the counter relays.

F. J. M.

14. R. L. Bisplinghoff, T. H. H. Pian & L. I. Levy, "A mechanical analyzer for computing transient stresses in airplane structures," *Jn. Appl. Mech.*, v. 17, 1950, p. 310–314.

If the point of suspension of a torsion pendulum is rotated in accordance with a function $\psi(t)$, the rotation θ of the pendulum satisfies the differential equation $\ddot{\theta} + \omega^2(\theta - \psi) = 0$ or $\ddot{\theta} + \omega^2\theta = \omega^2\psi(t)$. Thus a torsion pendulum permits an easy realization of this last equation. In the application described here, three such pendulums are set up with the angular output θ measured by wire strain gauge elements which when used in bridge circuits will yield a.c. voltages proportional to the angle of twist. A linear combination of these a.c. voltages may be obtained by using transformers and potentiometers.

F. J. M.

15. M. Doporto, "Theory and description of a gradient wind computer," Department of Industry and Commerce, Meteorological Service Geophysical Publications, v. 3, No. 1, 1950, 8 p., Stationery Office, Dublin, Eire.

The author devises a graphical computer based on a rather evident geometrical construction which solves for the real positive roots of certain quadratic equations expressing the meteorological gradient wind velocity. Many meteorologists are of the opinion that other approximations (such as $x = c$ for large $|b|$), or nomograms based on a different gradient wind equation, give the gradient velocity with sufficient accuracy for most purposes, considering the approximations inherent in the wind equations used and the errors existing in the forecasted values of the parameters involved.

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16. S. FIFER, A. S. MENGEL, G. W. EVANS, W. A. MERSMAN, L. BAUER, L. C. MERRILL, W. S. MELAHN, J. L. BURNSIDE, R. N. BRETOI, D. L. MARKUSEN, C. F. COOK, D. S. TEAGUE, R. D. GILPIN, H. H. GOODE, W. A. WHEATLEY, G. G. DEN BROEDER, R. R. BENNETT, A. S. FULTON, H. EHLERS, E. VOGEL, W. A. MCCOOL, P. BROCK, S. SHERMAN, A. H. MILLER, H. MEISSINGER, C. BECK, W. F. RICHMOND, *Symposium on REAC techniques*, Reeves Instrument Corporation, 215 E. 91 Street, New York 28, N. Y., 1951, 148 p.

Since the war, the use of REAC equipment has become widespread in certain fields. A REAC is an electromechanical differential analyzer, manufactured in "packages" consisting of seven feedback amplifiers and associated equipment to permit their use as integrators or summers, potentiometers and four servo units which are used as multipliers, dividers or resolvers. The purpose of "Symposium I" was to coordinate the experience of the many users of REAC equipment for their mutual benefit. (The symposium refers to "40 installations" many of which have more than one package.) The symposium took place on March 15, 16, 1951.

Session I. The first paper given here was by A. S. Mengel and concerns the application of the Rand REAC installation to an unusual type of problem in the calculus of variations concerning the motion of a dynamic system in which one of the end values is to be maximized. This problem has been studied mathematically in previous Rand reports by M. R. HESTENES who showed that its solution depended on the simultaneous solution of the equations of motion of the dynamic system and a set of Euler-Lagrange equations. The REAC equipment can be effectively utilized both to obtain solutions and also to determine the sensitivity of the solutions to various parameters which appear. In practical applications, this last is frequently a very important question.

The second paper of this session by G. W. EVANS is entitled "Some partial differential equations solvable by the REAC." A single first order partial differential equation can be solved by the method of characteristics, which is also applicable to certain linear partial differential equations of the second order in the hyperbolic and parabolic case. Two examples of these last with constant coefficients are given with detailed REAC circuits.

The third paper by W. A. MERSMAN is concerned with the solution of an equation in the form $L(x) = M(y)$ where L and M are linear differential operators with constant coefficients and y has a discontinuity at $t = 0$.

The fourth paper by L. BAUER & S. FIFER discusses the realizations of a polynomial in a complex variable z by either resolvers which generate sine and cosine or by obtaining these functions as the solution of differential equations. Either method is applicable to the REAC and such a realization of the polynomial can be used to locate the roots.

The concluding paper of this session is by L. C. MERRILL and is concerned with the evaluation of multidimensional integrals. These can be evaluated as a properly averaged one-dimensional integral on an ergodic path through the region of integration. The path is generated by sawtooth voltages of incompatible frequencies, and irregular regions of integration in the multidimensional space are handled by suitable on-off circuits. The range of the integration is extended by a direction reversing method.

Session II. W. S. MELAHN describes in the first paper of this session certain additions to the REAC equipment at Rand including an IBM plugboard method of set up, which permits one to retain the set up of an individual problem while the machine is used on a different problem, an operator's console, a digital read out and certain stabilized power supplies. By additional wiring to the parts of components, the Rand installation has attained a much greater flexibility and by additional checking and adjusting techniques a greater accuracy than the usual REAC installation.

The next paper by J. L. BURNSIDE advocates a number of changes in REAC design. One of these is the separation of heat producing non-manipulated components such as power supplies from the rest, a second is the substitution of a switching system for the patch bay set up. He also recommends the addition of certain equipment like high speed multipliers and printing readers for the servo units.

The third paper by R. N. BRETOI & D. L. MARKUSEN is concerned with the use of the REAC in the design of the Autopilot for an airplane. The design of such a device involves a number of stability considerations which are normally considered first in a simplified form by direct calculation and then all factors are studied in full generality on the REAC.

In testing airplane equipment and instruments, it is necessary to consider their performance under conditions simulating flight. This is usually done by mounting these on a rotating mount. C. F. COOK discusses the control of one such mount by REAC equipment and finds a region of stable control.

The next paper was by D. S. TEAGUE & R. D. GILPIN, "Preliminary investigation of the suitability of REAC for experimental curve fitting." The objective here is the determination of certain unknown coefficients in a differential equation, when a solution is given.

The concluding paper of this session by H. H. GOODE, W. A. WHEATLEY and G. G. DEN BROEDER is concerned with the problem of simulating N chance variables with a joint N dimensional distribution with given means and given covariance matrix, by analog equipment. This is accomplished by means of certain transformations on N variables which are the output of N noise generators, whose specifications are given.

F. J. M.

Session III. This session consisted of three papers on noise and frequency analysis and one on the solution of integral equations.

A. S. FULTON and R. R. BENNETT, "The generation and measurement of ultra low frequency random noise." In this article an electronic noise generator suitable for use with an electronic analog computer is described. The type of noise produced has a normal amplitude distribution. Its frequency spectrum is essentially flat at low frequencies with a half power point at 25 c.p.s. The mean value is practically zero and is obtained by passing the noise through a high pass filter. The frequency spectrum can be altered by passing the noise through various types of filters and the amplitude distribution can even be changed by suitable combinations of linear and non-linear filters. The (residual) mean value of the noise is obtained by integrating the voltage for a sufficiently long length of time. The mean square value can be determined essentially by half-wave recti-

fying the noise and then integrating. The spectrum of a particular noise can, of course, be computed from its autocorrelation function. In simulator work it is often simpler to pass the noise through various sharp band pass filters and measure the transmitted power.

H. EHLERS & E. VOGEL, "Simulation of noise in missile homing problems." (Only an abstract of this paper appears.) It is said to contain a description of the noise generator and the shaping of the output to simulate the noise as recorded in flight.

W. A. MCCOOL, "Frequency analysis by electronic analog methods." In this paper an electronic analog computer technique for evaluating Fourier transforms is described. The method is to consider the complex Fourier series expansion of period T of an arbitrary function $g(t)$, $g(t) = \sum_{-\infty}^{\infty} y_n(t) = \sum_{-\infty}^{\infty} F(\omega_n) \exp(j\omega_n t)$. For large values of T it is reasonable to assume $F(\omega_n)$ is a good approximation to the Fourier transform of $f(t)$. If, in the above equation, $F(\omega_n)$ is replaced by the Fourier transform $F(\omega) = \int_{-\infty}^{\infty} f(t) \exp(-j\omega t)\,dt$, there results, on dropping subscripts, $y(t) = \int_0^t f(\tau) \exp(j\omega(t-\tau))\,d\tau$. From this result, a linear differential equation on $y(t)$ (which can be solved by analog techniques) is derived. The method also applies to the evaluation of the inverse Fourier transform.

P. BROCK and S. SHERMAN, "Applications of REAC equipment to the solution of problems involving integral operators." In this paper practical methods (successfully used on REAC equipment) are given for the solution of typical problems involving integral operators. The method is by iteration using the "point-plot method." For example, the following problem was solved: $\varphi(s) = s + 2\int_0^1 st\varphi(t)\,dt$ where $\varphi_0(s) = s$ and (*) $\varphi_n(s) = s + 2\int_0^1 st\varphi_{n-1}(t)\,dt$. $\varphi_n(s)$ is determined from (*) using the previously determined φ_{n-1} curve as a function of t. To solve (*), $\varphi_n(s)$ is determined as a point function $[\varphi_n(s_i)]$ for a sequence of constant s values $[s_i]$. The REAC solves $\varphi_n(s_i, t) = s_i + 2\int_0^t s_i t\varphi_{n-1}(t)\,dt$ for each s_i. Note that $\varphi_n(s_i, 1) = \varphi_n(s_i)$. This function $\varphi_n(s_i)$ is plotted for each value of s_i on an output table.

Session IV. This session on numerical checks was devoted to a discussion by many of the members of the Symposium (under the chairmanship of P. Brock) of the general problem of determining the accuracy of REAC solutions. No papers were presented. Most of the remarks were of a qualitative nature.

Session V. This session consisted of five papers devoted to particular techniques used on the REAC for solving various special problems.

A. H. MILLER, "A REAC solution of a linear proton accelerator design problem." The mathematical problem considered in this paper is that of solving a number of systems of equations. Each system is composed of one linear and one non-linear differential equation. The initial conditions for the nth system are determined by the solutions of the $(n-1)$st system. The problem that arises in solving these equations on the REAC is that seven servos (necessary to generate $1/n$, cotangents, tangents, etc.) are needed while only four are available. By considering the numerical range of the variables involved, certain approximations and other circuit arrangements can be used. For example, $1/n$ can be generated by solving the differential equation, $dy/dn = -y^2/100$.

H. MEISSINGER, "Representation of functions of several variables on REAC equipment." One can represent a function of two variables by using

a three dimensional cam or a multi-wire pick-off device. Neither of these methods is particularly feasible. A method is described where certain functions can be broken up into sums and products of functions of a single variable. For example, it is shown that a certain function $z(x, y)$ which occurred in a recent problem on guided missile trajectory work could be approximated by $z(x, y) = f_3(x) - f_2(x)f_1(y)$. Approximate analytic expressions were determined for f_1, f_2 and f_3 and the result set up on the REAC.

C. BECK, "A method for solving problems on the REAC by use of transfer functions without passive networks." Some problems are stated directly in terms of the transfer function $H(s)$ of a linear system, that is, (*) $x_0(s) = H(s)x_i(s)$ where s is the Laplace transform variable and $H(s)$ is a rational function of s. A method is described whereby (*) can be solved without resorting to the original differential equation. The case where the system is not originally in the relaxed state (zero initial conditions) is also considered.

W. F. RICHMOND, "REAC techniques." Various techniques for treating continuous linear functions, continuous non-linear functions, discontinuous linear functions or linear functions with discontinuous derivatives, discontinuous non-linear functions or non-linear functions with discontinuous derivatives are discussed. Numerous circuit diagrams are included.

R. R. BENNETT, "The generation of straight line transfer relationships." The use of relay (sampling, clamping) amplifiers in generating straight line segments is described. Such techniques are necessary in the solution of non-linear problems on electronic analog computers.

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17. G. D. MCCANN & R. H. MACNEAL, "Beam vibration analysis with electric-analog computer," *Jn. Appl. Mech.*, v. 17, 1950, p. 13–26.

Four systems of partial differential equations in two independent variables x and t are solved by an analog method in which x is permitted to have only a small number of values so that the partial differential equations become an ordinary system on the variable t, which can be realized by an electric analog [*MTAC*, v. 3, p. 501–513]. The systems treated are linear of the second and fourth order on one, two or three unknown functions and the coefficients may depend on the variable x. The validity of the results also depends on an analogy: results from the analog are compared with analytic solutions in certain simple cases.

F. J. M.

18. A. D. MOORE, "Soap film and sandbed mapper techniques," *Jn. Appl. Mech.*, v. 17, 1950, p. 291–298.

19. MULTIX *Multiplying Sticks*. Larchmont, N. Y. $1.00.

This set consists of ten cardboard strips and two mimeographed pages of instruction. The strips are similar to the well known "Napier bones" for multiplying a number of any size by a one digit number. For each possible digit of the multiplicand a strip is given. When these are suitably arranged the product can be read off. However in these "Multix," the same strip can

be used for either 5 in the multiplicand or 55 or 555 and, consequently, only ten strips are needed. The price seems high and many people will find multiplication tables more useful, even if less intriguing.

F. J. M.

20. D. W. PENDERY, "604 electronic calculator control panels for multiply, divide, and square root on the card-programmed electronic calculator," IBM *Tech. Newsletter*, No. 1, 1950, p. 4–8.

W. BELL, "The 'DUZ' general-purpose control panel for the IBM type 604, electronic calculator," IBM *Tech. Newsletter*, No. 1, 1950, p. 9–11.

D. B. MACMILLAN & R. H. STARK, "A general-purpose 604 electronic calculator control panel for the card-programmed electronic calculator," IBM *Tech. Newsletter*, No. 1, 1950, p. 12–18.

B. OLDFIELD, "604 electronic calculator diagrams for the calculation of $\sin x$, $\cos x$, e^x, e^{-x}, $\sinh x$, and $\cosh x$ on the card-programmed electronic calculator," IBM *Tech. Newsletter*, No. 1, 1950, p. 19–25.

B. G. OLDFIELD, "The calculation of variance, covariance, and averages," IBM *Tech. Newsletter*, No. 2, 1951, p. 2–6.

B. G. OLDFIELD, "Numerical integration by Simpson's rule," IBM *Tech. Newsletter*, No. 2, 1951, p. 7–12.

W. D. BELL & AMY MCAVORY, "Graphical aids for determining shift codes," IBM *Tech. Newsletter*, No. 2, 1951, p. 13–15.

D. B. MACMILLAN & R. H. STARK, " 'Floating decimal' calculations on the IBM card-programmed electronic calculator," IBM *Tech. Newsletter*, No. 2, 1951, p. 16–26.

The introduction to *Technical Newsletter*, No. 1, reads as follows:

"Technical Newsletter Number 1 is the first of a series to be published by the Applied Science Department of the International Business Machines Corporation. The purpose of these Newsletters is to allow rapid interchange of information between computers concerning computational methods on IBM punched-card equipment.

Technical Newsletter Number 1 describes general purpose control panel diagrams which have been used successfully. Three of these diagrams are for the IBM Card-Programmed Electronic Calculator. The fourth diagram is for the IBM Type 604 Electronic Calculator."

In both numbers, the solution to the stated problems is given without mathematical discussion in the technical language of the IBM manuals. The "shift codes" referred to in the title of the BELL & MCAVORY paper deal with the position of the decimal point.

F. J. M.

21. H. PORITSKY, B. E. SELLS & C. E. DANFORTH, "Graphical, mechanical and electrical aids for compressible fluid flow," *Jn. Appl. Mech.*, v. 17, 1950, p. 37–46.

The flows studied in this paper are two dimensional with no vorticity and no viscosity. Consequently, they can be described by two potential functions, φ and ψ, by plotting the curves $\varphi = C_1$ and $\psi = C_2$ for equally

spaced values of C_1 and C_2. φ is the velocity potential and ψ is the streamline function and thus the two sets of curves intersect at right angles. The plot divides the given area into curvilinear rectangles, which in the case of incompressible flows are equisided. For the compressible case, Bernoulli's equation gives a relation between the sides of these curvilinear rectangles. The authors give a historical discussion of graphical methods in this problem and describe, with examples, a process of starting with the incompressible case and arriving at the desired result. For this purpose a number of mechanical aids are described. A four legged caliper is described which will give the proper size for the other side of the rectangle, when one side is given and a cam device for the same purpose is described. To permit a ready modification of the plot, the latter is realized by wires and pegs which have two holes bored at right angles through their sides. Lord Rayleigh has pointed out that the incompressible case can be plotted very effectively by using a double mesh of wires. For this purpose each peg has an additional pair of perpendicular holes making an angle of 45° with the first pair and the double mesh involves both the sides and the diagonals of the smaller squares. The authors also discuss the representation of the equations of fluid flow by means of a resistance network. Here the resistances are to be adjusted until the proper relation between potential and current flow is obtained.

F. J. M.

22. W. W. SOROKA, "Experimental aids in engineering design analysis," *Mech. Engineering*, v. 71, 1949, p. 831–837.

This is a survey article with a bibliography, covering certain analogue techniques for the solutions of partial and ordinary differential equations and algebraic equations.

23. M. A. WOODBURY, "Inverting modified matrices," Princeton Univ., Statistical Research Group, *Memorandum Report* 42, Princeton, New Jersey. 4 hectographed leaves.

This is a resumé, including the work of SHERMAN and MORRISON, of formulas for inverting matrices which have been obtained from a matrix with a known inverse by bordering or by modifying either an individual element or the elements in one row or column or both or in a number of rows and columns. "None of the proofs are difficult but considerable exploration was necessary to discover useful forms."

F. J. M.

NOTES

128. GAUSS TO GERLING ON RELAXATION.—In his recent book[7] on matrices ZURMÜHL traces the relaxation method[2] of solving linear equations back to DEDEKIND'S report of GAUSS'S lectures.[7] It is believed by some computers, however, that Gauss's method is a different one—namely, the related method given by SEIDEL.[3] In the interest of giving Gauss his proper credit as a proponent of relaxation, the following translation of a letter by Gauss[4] is offered. Moreover, in this same letter Gauss introduces a useful trick,[7] now generally forgotten, which seems to improve the convergence

of the relaxation process in a wide class of problems. The letter to C. L. GERLING dated at Göttingen, 26 Dec. 1823 is as follows.[1]

My letter got to the post office too late and was brought back to me. I am therefore opening it to add the practical directions on elimination. However, this method has many small special advantages which can be learned only by use.

As an example I take your measurements for Orber-Reisig.[5]

[The angle measurements communicated by Gerling (taken from a sheet found with Gauss's papers) were as follows, where 1 denotes Berger Warte, 2 Johannisberg, 3 Taufstein, and 4 Milseburg:

Repetitions	Angle
13	1.2 = 26°44′ 7″.423
28	1.3 = 77 57 53.107
26	1.4 = 136 21 13.481
50	2.3 = 51 13 46.600
6	2.4 = 109 37 1.833
78	3.4 = 58 23 18.161.]

I first make

$$[\text{the angle toward}]\,1 = 0,$$

whence from 1.3

$$3 = 77°57'53''.107$$

(I prefer this, because 1.3 has more weight than 1.2); then from

$$\left.\begin{array}{l} 13\,|\,1.2\,|\,2 = 26°44'\ 7''.423 \\ 50\,|\,2.3\,|\,2 = \qquad 6.507 \end{array}\right\} 2 = 26°44'\ 6''.696;$$

finally from

$$\left.\begin{array}{l} 26\,|\,1.4\,|\,4 = 136°21'13''.481 \\ 6\,|\,2.4\,|\,4 = \qquad 8.529 \\ 78\,|\,3.4\,|\,4 = \qquad 11.268 \end{array}\right\} 4 = 136°21'11''.641.$$

In order to improve the approximation still further, I find, from

$$\left.\begin{array}{l} 13\,|\,1.2\,|\,1 = -\ 0''.727 \\ 28\,|\,1.3\,|\,1 = \qquad 0 \\ 26\,|\,1.4\,|\,1 = -\ 1.840 \end{array}\right\} 1 = -\ 0''.855.$$

Since any common alteration of all directions is allowed when dealing only with relative position, I alter all four by $+0''.855$ and set

$$\begin{aligned} 1 &= \quad 0°\ 0'\ 0''.000 + a \\ 2 &= \quad 26\ 44\ \ 7.551 + b \\ 3 &= \quad 77\ 57\ 53.962 + c \\ 4 &= 136\ 21\ 12.496 + d. \end{aligned}$$

In the indirect[6] process it is very advantageous to ascribe a variation to *each* direction. You can easily convince yourself of this, if you compute the same example without this trick,[7] so that you lose the great convenience of always having as a control the sum of the absolute terms = 0. Now I form the four normal equations by the following scheme (in actual applica-

tion, and if the terms are more numerous, I separate the positive and negative terms): [The constants are expressed in units of the third decimal place.]

$ab - 1664$	$ba + 1664$	$ca + 23940$	$da - 25610$
$ac - 23940$	$bc + 9450$	$cb - 9450$	$db + 18672$
$ad + 25610$	$bd - 18672$	$cd - 29094$	$dc + 29094.$

The normal equations are therefore:

$$\begin{aligned} 0 &= + 6 + 67a - 13b - 28c - 26d \\ 0 &= - 7558 - 13a + 69b - 50c - 6d \\ 0 &= - 14604 - 28a - 50b + 156c - 78d \\ 0 &= + 22156 - 26a - 6b - 78c + 110d; \\ &\text{sum} = 0. \end{aligned}$$

In order to eliminate indirectly,[6] I note that, if 3 of the quantities a, b, c, d are set equal to 0, the fourth gets the largest value when d is chosen as the fourth.[8] Naturally every quantity must be determined from its own equation, and hence d from the fourth. I therefore set $d = -201$ and substitute this value. The absolute terms then become: $+5232$, -6352, $+1074$, $+46$; the other terms remain the same.

Now I let b take its turn, find $b = +92$, substitute, and find the absolute terms: $+4036$, -4, -3526, -506. And thus I continue until there is nothing more to correct. Of this whole calculation I actually write only the following table:[9]

	$d=-201$	$b=+92$	$a=-60$	$c=+12$	$a=+5$	$b=-2$	$a=-1$
$+ 6$	$+5232$	$+4036$	$+ 16$	-320	$+ 15$	$+41$	-26
$- 7558$	-6352	$- 4$	$+ 776$	$+176$	$+111$	-27	-14
-14604	$+1074$	-3526	-1846	$+ 26$	-114	-14	$+14$
$+22156$	$+ 46$	$- 506$	$+1054$	$+118$	$- 12$	0	$+26.$

Insofar as I carry the calculation only to the nearest 2000-th of a second, I see that now there is nothing more to correct. I therefore collect

$a=-60$	$b=+92$	$c=+12$	$d=-201$
$+ 5$	$- 2$		
$- 1$			
-56	$+90$	$+12$	-201

and add the common correction $+56$, whence

$$a = 0 \qquad b = +146 \qquad c = +68 \qquad d = -145;$$

therefore the values [of the directions] are

1	0° 0′ 0″.000
2	26 44 7.697
3	77 57 54.030
4	136 21 12.351.

Almost every evening I make a new edition of the tableau, wherever there is easy improvement.[10] Against the monotony of the surveying business, this is always a pleasant entertainment; one can also see immediately

whether anything doubtful has crept in, what still remains to be desired, etc. I recommend this method to you for imitation. You will hardly ever again eliminate directly, at least not when you have more than 2 unknowns. The indirect procedure can be done while half asleep, or while thinking about other things.[11]

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[1] The preparation of this translation was sponsored (in part) by the Office of Naval Research. The author wishes to acknowledge the assistance of Drs. FRITZ JOHN and OLGA TAUSSKY-TODD.

[2] See R. V. SOUTHWELL, *Relaxation Methods in Engineering Science*. Oxford, 1940, and L. FOX, "A short account of relaxation methods," *Quart. Jour. Mech. Appl. Math.*, v. 1, 1948, p. 253–280.

[3] Ludwig Seidel, "Über ein Verfahren, die Gleichungen, auf welche die Methode der kleinsten Quadrate führt, sowie lineare Gleichungen überhaupt, durch successive Annäherung aufzulösen," *Akad. Wiss., Munich, mat.-nat. Abt. Abhandlungen*, v. 11, 1874, p. 81–108. Although Seidel's process is frequently called the Gauss-Seidel process, I know of no place where Gauss mentions it.

[4] The letter is in Gauss's *Werke*, v. 9, 1903, p. 278–281. It is probably the letter referred to in the footnote, p. 257, of E. T. WHITTAKER and G. ROBINSON, *The Calculus of Observations*, 1st edit., London and Glasgow, 1924, as a source of their example of relaxation (p. 257–8).

[5] Words within brackets are translations of inserts by L. KRÜGER, who prepared volume 9 of Gauss's *Werke*.

[6] 'Indirect elimination' was Gauss's term for his iterative process of solving the normal equations. It later came to denote any iterative process for solving linear equations.

[7] The symmetric treatment of the unknowns is an essential idea in this and other letters. Here Gauss mentions only its advantage as a device which sets up a column-sum check to detect errors. In later letters (cf., e.g., Gauss to Gerling, 19 January 1840, *Werke*, vol. 9, p. 250–3) Gauss is convinced that the symmetric treatment of all unknowns yields normal equations whose iterative solution converges significantly faster.

The trick is later described by: Christian Ludwig Gerling (recipient of the letter), *Die Ausgleichungsrechnung der practischen Geometrie*. Hamburg and Gotha, 1843 (p. 157–8, p. 163, p. 386, p. 390); by R. Dedekind, "Gauss in seiner Vorlesung über die Methode der kleinsten Quadrate," *Festschrift zur Feier des 150-jährigen Bestehen der königlichen Gesellschaft der Wissenschaften zu Göttingen*. Berlin, 1901 (pp. 45–59) and *Gesammelte Mathematische Werke*. V. 2, 1931, p. 293–306; and by R. Zurmühl, *Matrizen*, Berlin, 1950, p. 280–282. (Zurmühl is wrong, however, in stating that the trick will improve the convergence for all badly conditioned systems of equations.)

For some discussion of when and why the trick may be expected to improve the convergence of iterative processes for solving linear equations, see George E. Forsythe and THEODORE S. MOTZKIN, "An extension of Gauss's transformation for improving the condition of systems of linear equations," multilithed typescript at the National Bureau of Standards, Los Angeles.

[8] Gauss is here using a method which relaxers recommend: liquidating "that residual . . . which requires the largest 'displacement' " (Fox, *op. cit.*, p. 256).

[9] Study of the table shows the algorithm to be precisely the relaxation method described by Fox (*op. cit.*, p. 255–6). In comparing Gauss's and Fox's presentations, we note that Gauss uses the method mentioned in note 8, and that he "liquidates the residuals" only approximately at each stage, as recommended by Fox (*op. cit.*, p. 257–8) to save unnecessary arithmetic. Gauss does not, however, propose "under-relaxation" or "over-relaxation," as Fox does. On the other hand, Gauss's trick mentioned in note 7 is not mentioned by Fox, although it is extremely helpful in many common problems.

[10] Gauss wrote, "Fast jeden Abend mache ich eine neue Auflage des Tableaus, wo immer leicht nachzuhelfen ist." It is not clear what he meant.

[11] Here one must bear in mind Gauss's gift for mental arithmetic!

QUERIES

49. FIRST USE OF THE TERM HAVERSINE AND THE FIRST TABLE OF HAVERSINES.—The editors of the great Oxford *New English Dictionary* (*NED*) endeavoured to bring together quotations exhibiting the first use

in English (so far as known) of each word. In the case of Haversine the quotation is from F. G. D. BEDFORD (1838–1913), *The Sailor's Pocket Book . . .*, second edition, London, 1875, p. 381. A reference can, however, be given to a work published 40 years earlier. In the *Dictionary of National Biography*, v. 10, 1887, J. K. LAUGHTON's biography of JAMES INMAN (1776–1859), the following sentences occur: "In 1821 appeared his well-known book *Navigation and Nautical Astronomy for the Use of British Seamen*, with accompanying tables. In the third edition (1835) he introduced a new trigonometrical function, the half-versine, or haversine, the logarithms of which were added in the tables." Hence here is a new reference, to replace that of Bedford, for the editors of *NED*. The earliest table of haversines which I have found is the rather elaborate one in JAMES ANDREWS', *Astronomical and Nautical Tables*, London, 1805 (see *MTAC*, v. 1, p. 422). Can any reader report earlier use of the word haversine, or an earlier table of the function? In what Libraries may the 1835 editions of Inman's works (*Navigation*, and *Tables*) be found?

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CORRIGENDA

V. 3, p. 497, l. −5, *for* b^m *read* b^{km}.
V. 4, p. 124, l. −2, *for* 96–97 *read* 496–497.
V. 5, p. 25, 854, l. 7, *for* 0(.001)25, *read* 0(.001)16(.01)25.

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